Midterm Exam 2. E&M, Spring 2015. Instructor: E. Dagotto. Delivered: March 27. Deadline: April 2 (at the start of class). Maximum # of points: 20.

(1) Magnetostatics (10 points). This is an "inverse" magnetic shielding problem. Consider a spherical shell of inner radius a and outer radius b, made of a linear material with permeability μ. Inside and outside the shell there is vacuum. Assume that at the center of the shell there is a tiny magnetic dipole of magnitude m pointing along the z axis that produces a magnetic scalar potential $\Phi_M = m \cos(\theta)/ (4 \pi r^2)$, where r and θ are the usual spherical coordinates. There are no currents anywhere in the problem.

(a) Find the magnetic scalar potential Φ_M all over space. Assume that only l=1 contributes in the expansions because the only source of fields is the l=1 dipole, i.e. no need to show that the coefficients for l different from 1 all cancel. Write the 4x4 system of equations for the coefficients.

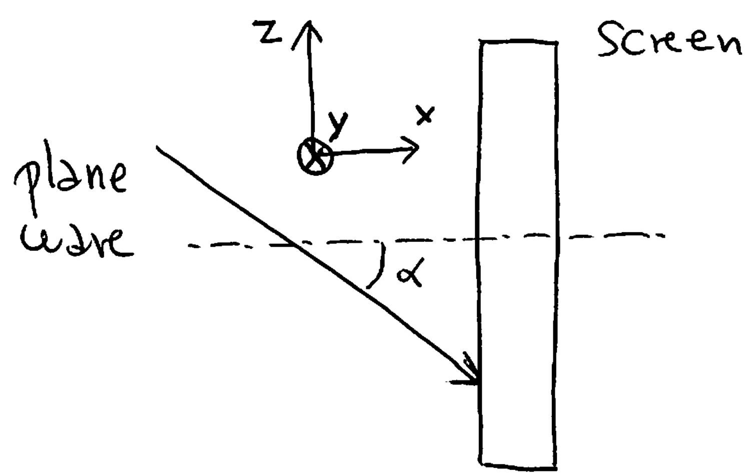
(b) Solve the 4x4 system of equations found in (a) for the l=1 coefficients via *Mathematica* or equivalent software if you wish. Or you can do this task by hand if you like.

(c) From the result in (b), write the potential Φ_M outside the shell ($r > b$) keeping only the leading term at large μ. Then find the magnetic field **B** and show that outside the shell it has the same dipolar form as the center dipole but with a magnitude that vanishes as $1/\mu$ (thus, this is indeed an inverse magnetic shielding problem).

(d) From the result in (b), show that the magnetic field **B** inside the shell ($a < r < b$) remains finite even in the limit $\mu \rightarrow \infty$.

All this means that in the $\mu \rightarrow \infty$ limit the lines of magnetic field that are produced by the magnetic dipole at the center are entirely confined inside $r < b$ (shell and center hole), but totally vanish outside.

(2) Maxwell stress tensor (5 points). This is a simple generalization of the problem solved in class, and in the homework, of a plane wave that hits a flat screen. The entire wave is absorbed by the screen (no reflection or transmission). See figure.



The electric field is polarized along the y axis with magnitude $E_0 \cos(\omega t)$, where ω is the frequency of oscillation. Note that there is no coordinate **x** dependence just for simplicity. The magnetic field component has an amplitude and direction that can be deduced from the electric field component based on the properties of plane waves.

(a) Construct the 3x3 Maxwell stress tensor, keeping the time dependence explicitly.

(b) Find the force on the screen. In this case take a time average.

(c) Confirm that all the results reduce to those found in class and homework for $\alpha = 0$.

(3) A "strange" metal (5 points). Consider a material, that we call "S", that admits having a current density **J**. The material is also linear and has a permeability μ. There are no sources of electric fields in the problem and the **E** and **D** fields can be considered zero all over. The material is metallic, but does not satisfy the phenomenological Ohm's law $\mathbf{J}=\sigma\mathbf{E}$ (we just said $\mathbf{E} = 0$!), but instead satisfies the entirely phenomenological London equation $\mathbf{J} = -\mathbf{A}/K$ where $K>0$ is a constant and **A** is the usual vector field. The problem is time independent i.e. we are in a steady state.

(a) Using the only two active Maxwell equations under these circumstances and the relation between **B** and **A**, find the second order differential equation satisfied by the magnetic field **B** (not the equation for **A** but directly the one satisfied by **B**).

(b) Consider a planar interface between "S" and vacuum. On the vacuum side, consider a fixed static magnetic field $\mathbf{B}_0$ pointing say along the x axis which is parallel to the interface. Using the equation found in (a), find the value of the field **B** as it penetrates into the material "S". What is the characteristic length for the field suppression in terms of μ and K? This length is called the London penetration length and the material "S" is actually a superconductor. The suppression that you found is the famous Meissner effect.