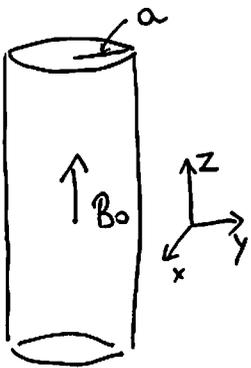


The magnetic field of an infinite solenoid is constant and homogeneous inside the solenoid, pointing along the z-axis. Let us call this field B_0 ($B_0 = \mu_0 \left(\frac{N}{l}\right) I$ to be more precise).

2

(a)
There is no electric field. Thus, to construct the stress tensor we use:



$$E_x = E_y = E_z = 0$$

$$B_x = B_y = 0$$

$$B_z = B_0 \text{ inside and } 0 \text{ outside}$$

$$T_{\alpha\beta} = \epsilon_0 c^2 \left[B_\alpha B_\beta - \frac{1}{2} (\vec{B} \cdot \vec{B}) \delta_{\alpha\beta} \right]$$

$$T_{\alpha\beta} = 0 \text{ if } \alpha \neq \beta$$

$$T_{xx} = \epsilon_0 c^2 \left(-\frac{1}{2}\right) B_0^2 \stackrel{c^2 = \frac{1}{\mu_0 \epsilon_0}}{=} -\frac{1}{2\mu_0} B_0^2$$

$$T_{yy} = \epsilon_0 c^2 \left(-\frac{1}{2}\right) B_0^2 = -\frac{1}{2\mu_0} B_0^2$$

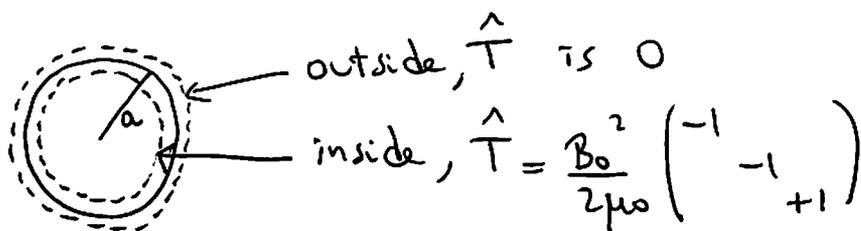
$$T_{zz} = \epsilon_0 c^2 \left[B_0^2 - \frac{1}{2} B_0^2 \right] = +\frac{1}{2\mu_0} B_0^2$$

Then, as a 3×3 matrix \hat{T} is:

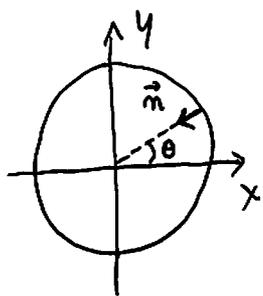
$$\hat{T} = \frac{B_0^2}{2\mu_0} \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & +1 \end{pmatrix} \text{ with } B_0 = \mu_0 \left(\frac{N}{l} \right) I$$

(b)

Consider the imaginary volume indicated by dotted line, seen from above (i.e. from +z-axis)



This is similar to the homework problem about the first screen.



$\vec{n} = -\cos\theta \hat{e}_x - \sin\theta \hat{e}_y$ is the unit vector to be used in the inside of the imaginary volume.

Then, $\oint_S \sum_{\beta} T_{\alpha\beta} m_{\beta} da$ will be nontrivial for $\alpha = x$ and y .

Consider $\alpha = x$:

$$F_x = \oint_S \sum_{\beta} T_{x\beta} m_{\beta} da = \oint_S T_{xx} m_x da = \oint_S \left(\frac{-B_0^2}{2\mu_0} \right) (-\cos\theta) \underbrace{dz d\theta}_{\text{cylindrical coordinates}}$$

$$= \frac{B_0^2 a}{2\mu_0} \int_{-L}^L dz \int_0^{2\pi} \cos\theta d\theta = 0$$

\uparrow
 assuming $2L$
 is the length

$\underbrace{\int_0^{2\pi} \cos\theta d\theta}_{\sin\theta \Big|_0^{2\pi} = 0} = 0$

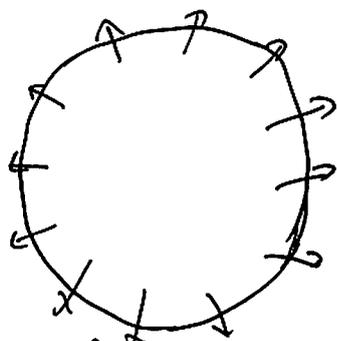
Consider $x = y$:

$$F_y = \oint_S \sum_{\beta} T_{y\beta} n_{\beta} da = \oint_S T_{yy} n_y da = \oint_S \left(-\frac{B_0^2}{2\mu_0}\right) (-\sin\theta) a dz d\theta$$

$$= \frac{B_0^2 a}{2\mu_0} \int_{-L}^L dz \int_0^{2\pi} \sin\theta d\theta = 0$$

$\underbrace{\int_0^{2\pi} \sin\theta d\theta}_{-\cos\theta \Big|_0^{2\pi} = 0} = 0$

Then, the net force is 0 which is reasonable by symmetry, as the sketch indicates:

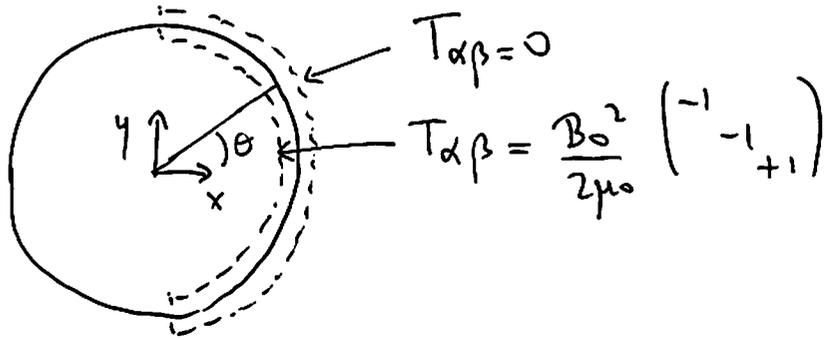


The azimuthal symmetry of the problem leads to the cancellation of the net force.

Thus, the total force cancels but not the force acting on a fraction of the area.

(c)

To verify this idea consider just one half of the solenoid as indicated with dotted lines:



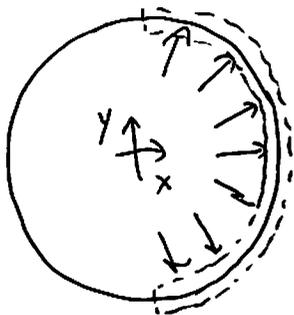
Repeating the previous calculation we arrive to:

$$F_x = \frac{B_0^2 a}{2\mu_0} (2L) \int_{-\pi/2}^{\pi/2} \cos\theta d\theta = \frac{B_0^2 a (2L)}{2\mu_0} \left. \sin\theta \right|_{-\pi/2}^{\pi/2} = \frac{B_0^2 a (2L) \cdot 2}{2\mu_0} \neq 0$$

and

$$F_y = \frac{B_0^2 a}{2\mu_0} (2L) \int_{-\pi/2}^{\pi/2} \sin\theta d\theta = \frac{B_0^2 a (2L)}{2\mu_0} \left. (-\cos\theta) \right|_{-\pi/2}^{\pi/2} = 0$$

This is reasonable by symmetry as the sketch indicates:



Net force along x
Cancellation along y

Thus, the material that the solenoid is made of must be able to stand the pressure that the interior magnetic field exerts on the walls.

In practice, this can be a problem for large magnetic fields.

In summary of part (c), the force per unit length (i.e. divided by " $2L$ ") is

$$\frac{F_x}{(2L)} = \frac{B_0^2 a}{\mu_0}, \quad \frac{F_y}{(2L)} = 0, \quad \frac{F_z}{(2L)} = 0$$