FINAL Mid-term exam. Electricity and Magnetism, Spring 2012.

Instructor: Elbio Dagotto. April 24, 2012. Deadline: May 1, 2012 at noon. Drop the solutions in professor's mailbox at the Physics Dept.

(1) A thin spherical shell of radius a has a potential fixed to $V_0\cos\theta$, where θ is the standard angle used in spherical coordinates.

(a) Find the potential $\phi(r,\theta)$ outside the shell, using standard separation of variables. Calculate the equivalent electric dipole vector **p** that produces this potential ϕ. Thus far, this is just a simple problem of electrostatics.

(b) Assuming now the standard $e^{-i\omega t}$ time dependence for the electric dipole **p**, calculate the magnetic field **H** and electric field **E** in the radiation zone. For (b), and also (c) and (d) below, you can use the generic formulas in the book of Jackson for an oscillating dipole, no need to derive again those formulas.

(c) Get the time-averaged power radiated per unit solid angle i.e. $dP/d\Omega$. Express the results in terms of ω, θ, a, V_0, ε_0, and c.

(d) Get the total power radiated P.

(2) This is problem 11.16 of Griffiths, plus a few short questions. Repeat the same analysis as in Example 11.3 for the case of collinear velocity and acceleration of a charged particle, but now for the case where velocity and acceleration are perpendicular. Choose your axes such that the velocity **v** is along the z-axis and the acceleration **a** along the x-axis. Start the analysis with Eq. (11.72) and prove that $dP/d\Omega$ is indeed the result given in the text of Problem 11.16 of Griffiths i.e.

$$\frac{dP}{d\Omega} = \frac{\mu_0 q^2 a^2}{16\pi^2 c}\frac{\left[(1-\beta\cos\theta)^2 - (1-\beta^2)\sin^2\theta\cos^2\phi\right]}{(1-\beta\cos\theta)^5}.$$

No need to calculate P. Extra brief questions: (a) Is the final result the same for (−**a**) and for **a**? I.e. does anything change if I invert the direction of the acceleration keeping the velocity the same? (b) For the case of the angle $\phi=0$, find the value of the angle θ where $dP/d\Omega$ cancels. (c) Calculate the ratio of the $dP/d\Omega$'s for the cases $\theta=0$ and $\theta=\pi$. How large is this ratio for $\beta=0.9$?