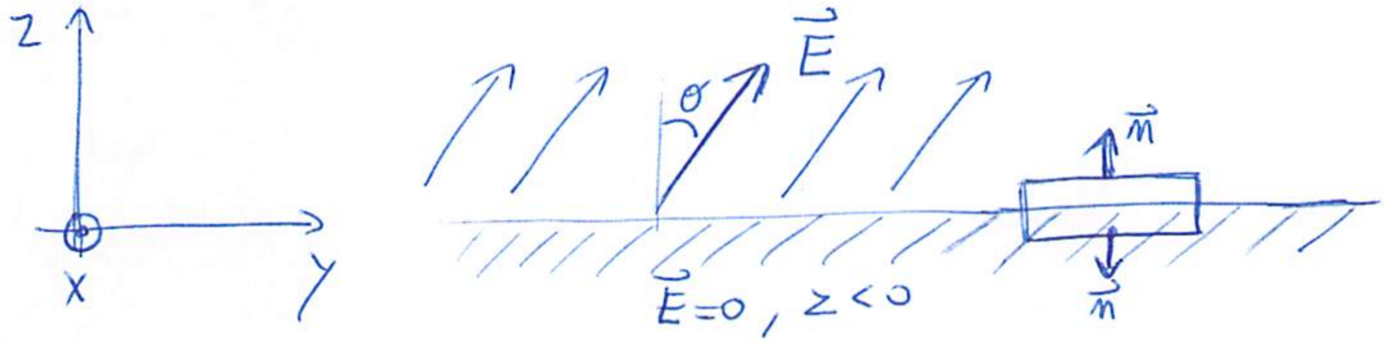
(a)

$$T_{\alpha\beta} = \epsilon_0 \left[E_\alpha E_\beta - \frac{1}{2}(\vec{E}\cdot\vec{E})\delta_{\alpha\beta} \right]$$

if magnetic fields are zero.



$$\vec{E} = E_0 \cos\theta\, \hat{e}_z + E_0 \sin\theta\, \hat{e}_y$$

$$\vec{E}\cdot\vec{E} = E_0^2(\cos^2\theta + \sin^2\theta) = E_0^2$$

$$T_{zz} = \epsilon_0 \left[E_z^2 - \frac{1}{2}(\vec{E}\cdot\vec{E}) \right] = \epsilon_0 \left[E_0^2 \cos^2\theta - \frac{1}{2}E_0^2 \right]$$

$$T_{yy} = \epsilon_0 \left[E_y^2 - \frac{1}{2}(\vec{E}\cdot\vec{E}) \right] = \epsilon_0 \left[E_0^2 \sin^2\theta - \frac{1}{2}E_0^2 \right]$$

$$T_{xx} = \epsilon_0 \left[E_x^2 - \frac{1}{2}(\vec{E}\cdot\vec{E}) \right] = \epsilon_0 \left[0 - \frac{1}{2}E_0^2 \right]$$

$$T_{xy} = T_{xz} = 0$$

$$T_{yz} = \epsilon_0 E_y E_z = \epsilon_0 E_0^2 \sin\theta\cos\theta.$$

$$\hat{n} = \hat{e}_z$$

$$\sum_{\beta} T_{\alpha\beta} n_{\beta} = T_{\alpha z} n_z \neq 0 \text{ for } \alpha = z \text{ and } y$$

Consider $\alpha = z$

$$T_{zz} n_z = \epsilon_0 E_0^2 \left(\cos^2\theta - \tfrac{1}{2}\right)$$

$$T_{yz} n_z = \epsilon_0 E_0^2 \sin\theta \cos\theta$$

$$F_z = \text{Area} \cdot \epsilon_0 E_0^2 \left(\cos^2\theta - 1/2\right)$$

$$F_y = \text{Area} \cdot \epsilon_0 E_0^2 \sin\theta \cos\theta$$

Use trigonometric identities

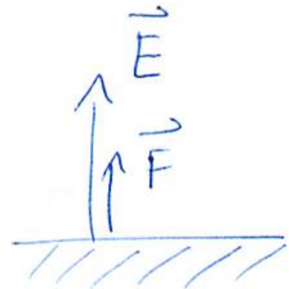
$$2\sin\theta \cos\theta = \sin(2\theta)$$

$$\cos 2\theta = 2\cos^2\theta - 1$$

$$\frac{F_z}{\text{Area}} = \epsilon_0 E_0^2 \cancel{z} \frac{\cos 2\theta}{2} \qquad \text{(pressure)}$$

$$\frac{F_y}{\text{Area}} = \epsilon_0 E_0^2 \frac{\sin(2\theta)}{2} \qquad \text{(shear)}$$

(b) At $\theta = 0$, the electric field is
and (c) vertical and the force only vertical
as it has to be:

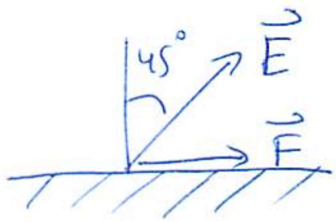


$$\frac{F_z}{\text{Area}} = \epsilon_0 E_0^2 \frac{1}{2}$$

$$\frac{F_y}{\text{Area}} = 0$$

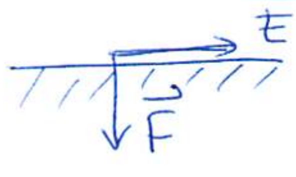
At $\theta = \pi/4$, the result is more interesting

$$\frac{F_z}{\text{Area}} = 0 \quad , \quad \frac{F_y}{\text{Area}} = \epsilon_0 E_0^2 \frac{1}{2}$$



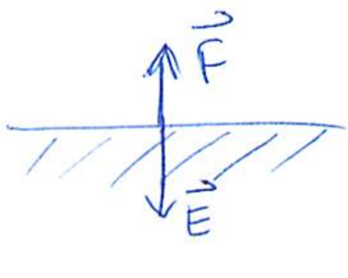
Only shear!

At $\theta = \pi/2$, $\dfrac{F_z}{\text{Area}} = -\dfrac{\epsilon_0 E_0^2}{2}$, $\dfrac{F_y}{\text{Area}} = 0$



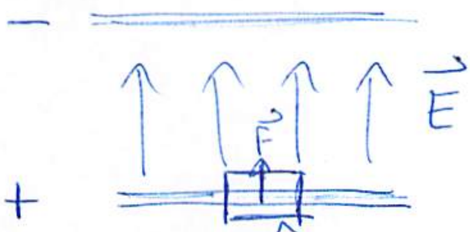
"electric pressure"

At $\theta = \pi$, $\dfrac{F_z}{\text{Area}} = \epsilon_0 E_0^2 \dfrac{1}{2}$, $\dfrac{F_y}{\text{Area}} = 0$



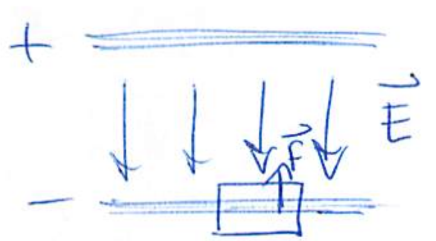
The force is the same whether $\vec{E}$ points "in" or "out", because only $(\vec{E})^2$ matters. It is a pressure.

Imagine an ideal capacitor:



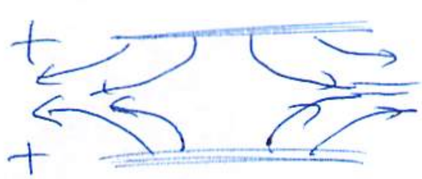
the lower plate is attracted to the upper one, thus force goes up as we found for $\theta = 0$.

To get $\theta = \pi$, we have to invert the polarity:



Force still points up because we are still attracted by the upper plate

If the charges are equal we are not representing the same physics as in the problem because fields are not constant



For $\theta = \pi/2$ result, just consider a small disk at the edge of a finite capacitor. The disk will be pushed to the left because the $\vec{E}$ wishes to expand