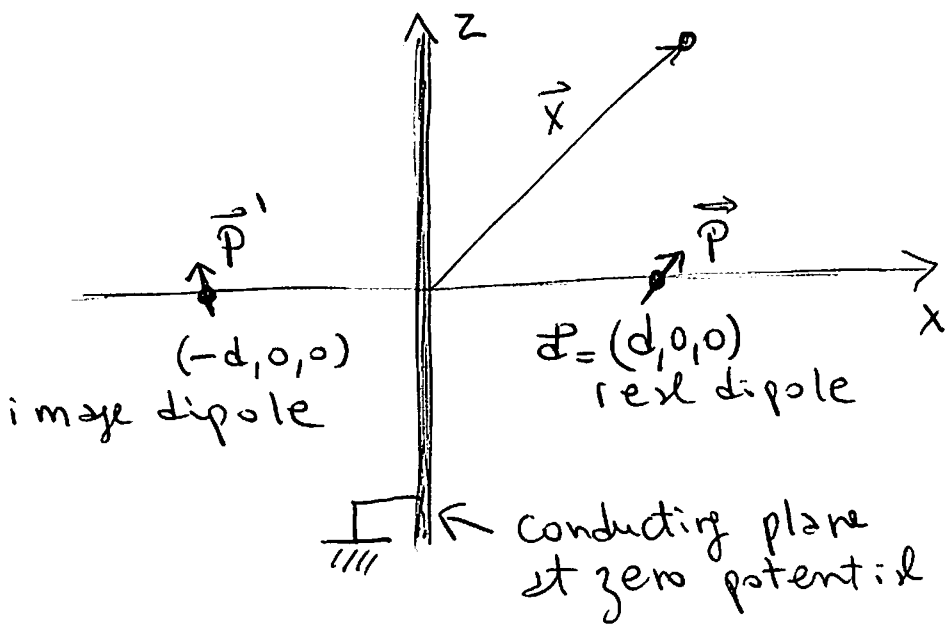
(a) The scalar potential ϕ caused by a dipole at the origin of coordinates is $\phi = \frac{1}{4\pi\epsilon_0} \cdot \frac{\vec{p}\cdot\vec{x}}{|\vec{x}|^3}$.

Consider this dipole now at position $(d,0,0)$ as in the sketch:



We will try with a dipole image at the same distance from the plane as the real dipole, with a vector $\vec{p}\,'$ to be determined.

The potential at an arbitrary location $\vec{x}$ is

$$\phi = \frac{1}{4\pi\epsilon_0}\left[\frac{\vec{p}\cdot(\vec{x}-\vec{d})}{|\vec{x}-\vec{d}|^3} + \frac{\vec{p}\,'\cdot[\vec{x}-(-\vec{d})]}{|\vec{x}-(-\vec{d})|^3}\right] =$$

$$= \frac{1}{4\pi\epsilon_0}\left[\frac{\vec{p}\cdot(\vec{x}-\vec{d})}{|\vec{x}-\vec{d}|^3} + \frac{\vec{p}\,'\cdot(\vec{x}+\vec{d})}{|\vec{x}+\vec{d}|^3}\right]$$

Now let us study a point $\vec{x}$ on the plane. For instance, let us say $\vec{x} = \rho\,\hat{e}_z = (0,0,\rho) = \vec{\rho}$

We want to find a $\vec{p}\,'$ such that $\phi = 0$ in the conducting plane.

$$\phi_{\text{plane}} = \frac{1}{4\pi\epsilon_0}\left[\frac{\vec{p}\cdot(\vec{\rho}-\vec{d})}{|\vec{\rho}-\vec{d}|^3} + \frac{\vec{p}\,'\cdot(\vec{\rho}+\vec{d})}{|\vec{\rho}+\vec{d}|^3}\right]$$

$$|\vec{\rho}-\vec{d}| = \sqrt{(\vec{\rho}-\vec{d})\cdot(\vec{\rho}-\vec{d})} = \sqrt{\rho^2+d^2}$$

$$\vec{\rho}\cdot\vec{d} = 0$$

$$|\vec{\rho}+\vec{d}| = \sqrt{(\vec{\rho}+\vec{d})\cdot(\vec{\rho}+\vec{d})} = \sqrt{\rho^2+d^2}$$

$$\vec{p}\cdot(\vec{\rho}-\vec{d}) = p_z\rho - p_x d$$

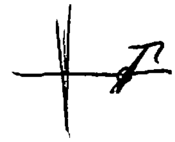
$$\vec{p}\,'\cdot(\vec{\rho}+\vec{d}) = p_z'\rho + p_x' d$$

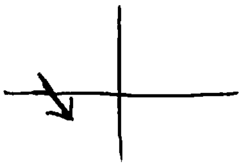
$$\phi_{\text{plane}} = \frac{1}{4\pi\epsilon_0}\frac{1}{(\rho^2+d^2)^{3/2}}\left[p_z\rho - p_x d + p_z'\rho + p_x' d\right]$$

For ϕ_{plane} to cancel for any ρ we need:

$$\boxed{\begin{aligned} p_z &= -p_z' \\ p_x &= p_x' \end{aligned}}$$

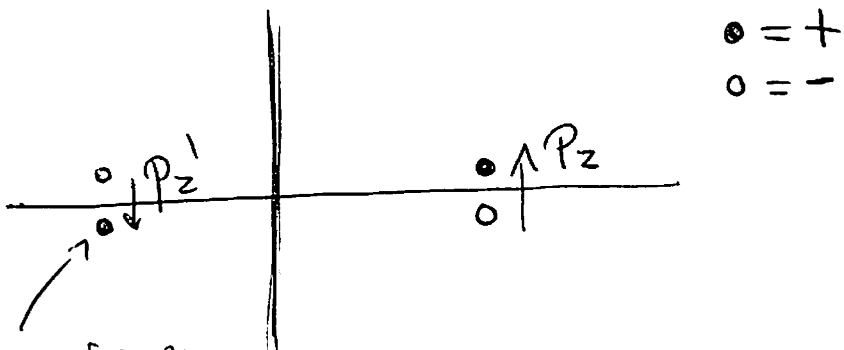
So one component has the same sign but the other opposite sign

If $\vec{p}$ is like  , then $\vec{p}\,'$ is like

 , same x component but opposite z component.

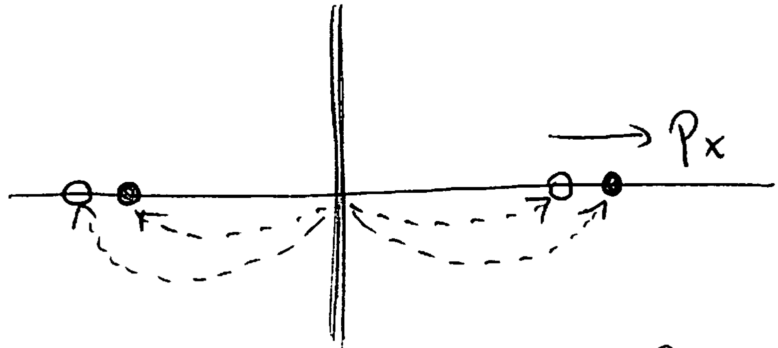
(b)

Let us try to understand this based on thinking of a dipole as the limit of two point like charges. Consider first the case of $\vec{p}$ pointing along the z axis:



the mirror image of a positive charge is negative and vice verra. Clearly the mirror images lead to a dipole pointing the other way as p_z, as in the math of previous pages; $p_z = -p_z'$.

Consider now a $\vec{p}$ pointing along the x-axis:

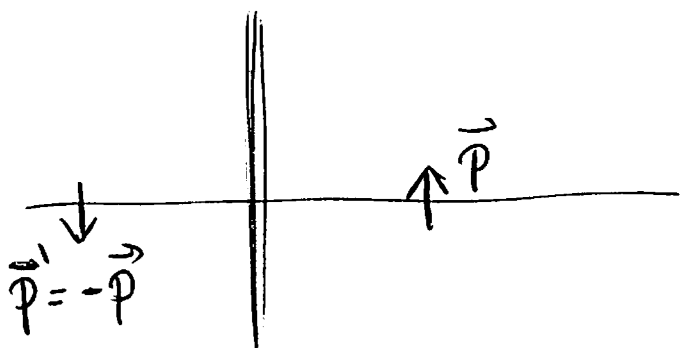


Now the mirror charges form a dipole pointing in the same direction as p_x i.e.

$$p_x' = p_x$$

This confirms the mathematical results.

(c) To get a quadrupolar component at large distances we want a net zero dipole. This is achieved only if $\vec{p}$ points along the z-axis, so that $\vec{p}'$ becomes $-\vec{p}$.

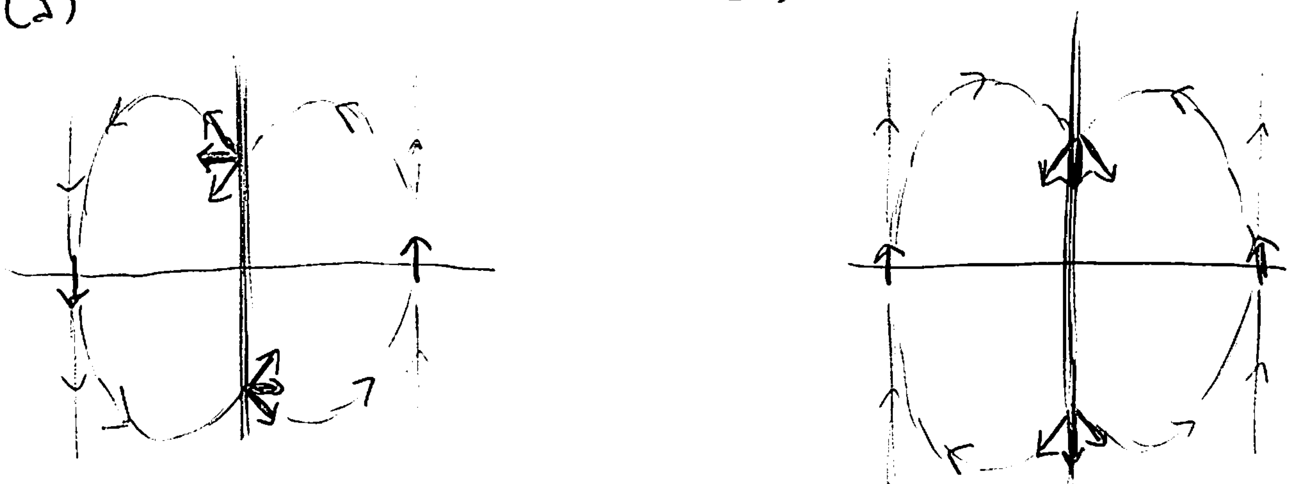


Otherwise there is a net dipole moment at large distances.

Extra information. $\vec{E}$ fields

For two dipoles pointing vertically in opposite directions as shown below, the electric field is perpendicular to plane, as it should. This is shown in (a) for dipoles of opposite directions. In (b), the same direction is assumed and now $\vec{E}$ at the plane is incorrectly tangent to the plane.

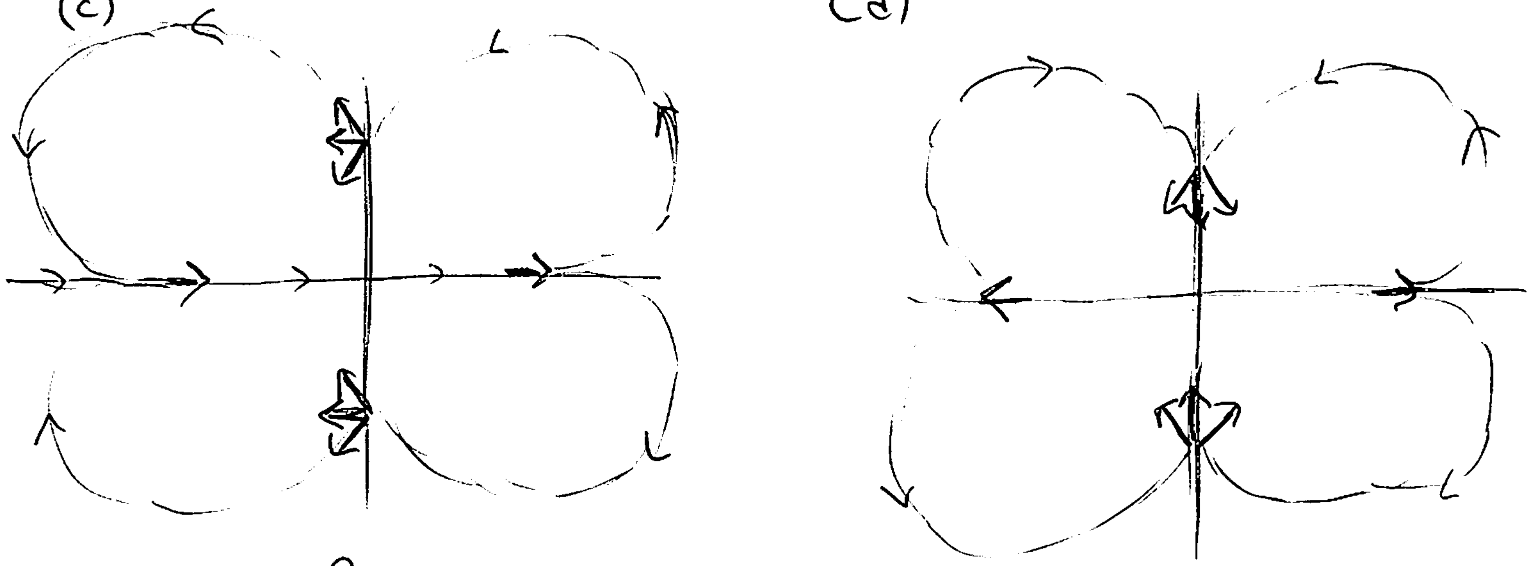
(a) (b)



Correct Incorrect

Same for "horizontal" dipoles as shown in (c) and (d).

(c) (d)



Correct Incorrect