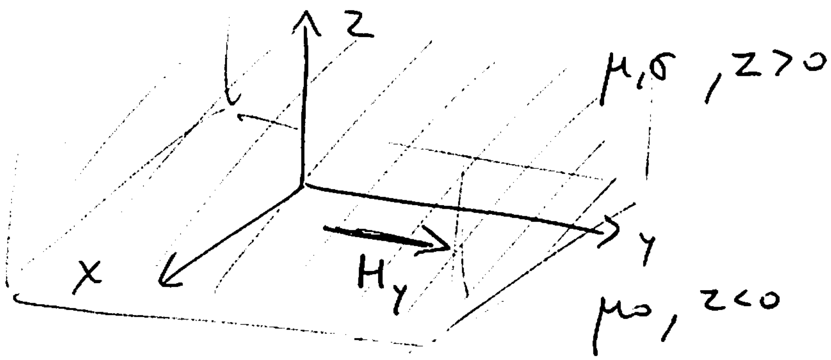


$H_y = H_0 \sin\omega t$ at $z = 0 - \varepsilon$ (or 0^-)

By continuity of the tangential component of $\vec{H}$ at an interface (i.e. $\hat{n} \times (\vec{H}_2 - \vec{H}_1) = 0$) then at $0+\varepsilon$ (or 0^+), H is the same:

$$H_y(z=0^+) = H_0 \sin(\omega t)$$

Following the same steps in the book or in the lecture, we arrive to the "difussion equation":

$$\nabla^2 \vec{A} = \mu\sigma \frac{\partial \vec{A}}{\partial t}$$

inside the upper plane.

From the discussion in the lecture, the vector potential that produces a magnetic field pointing along the y direction is

$$A_z = -\mu H(z,t)\, x \ , \ A_x = A_y = 0$$

because $\nabla \times \vec{A} = \begin{vmatrix} \hat{e}_x & \hat{e}_y & \hat{e}_z \\ \nabla_x & \nabla_y & \nabla_z \\ 0 & 0 & -\mu H(z,t)x \end{vmatrix} =$

$$= 0.\hat{e}_x + \mu H(z,t)\,\hat{e}_y + 0.\hat{e}_z$$

$$\vec{H} = \mu^{-1}\vec{B} = \frac{\nabla \times \vec{A}}{\mu} = \frac{\cancel{\mu}}{\cancel{\mu}} H(z,t)\,\hat{e}_y$$, which is correct for sure at $z = 0^+$.

Since the three components of $\vec{A}$ satisfy independent equations $\nabla^2 A_i = \mu\sigma \frac{\partial A_i}{\partial t}$ $(i = x, y, z)$

and since $A_x = A_y = 0$ at $z = 0^+$, then they are zero everywhere. Thus, the whole effort reduces to solving

$$\nabla^2 A_z = \mu\sigma \frac{\partial A_z}{\partial t}$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) A_z = \mu\sigma \frac{\partial A_z}{\partial t}$$

$$\Big(\underbrace{\frac{\partial^2}{\partial x^2}}_{0} + \underbrace{\frac{\partial^2}{\partial y^2}}_{0} + \frac{\partial^2}{\partial z^2}\Big)\left(-\mu H(z,t)x\right) = \mu\sigma(-\mu x)\frac{\partial H(z,t)}{\partial t}$$

$$-\mu x \frac{\partial^2}{\partial z^2} H(z,t) = -\mu^2\sigma x \frac{\partial H(z,t)}{\partial t}$$

$$\frac{\partial^2 H(z,t)}{\partial z^2} = \mu\sigma \frac{\partial H(z,t)}{\partial t}$$

Propose $H(z,t) = h(z)\, \underbrace{Re\left(i e^{-i\omega t}\right)}$

$$Re\left(i\left(\cos(\omega t) - i\sin(\omega t)\right)\right)$$
$$= \sin(\omega t).$$

$$\frac{\partial^2}{\partial z^2}\left(h(z)\, i e^{-i\omega t}\right) = \mu\sigma \frac{\partial}{\partial t}\left(h(z)\, i e^{-i\omega t}\right)$$

$$i e^{-i\omega t} \frac{\partial^2 h(z)}{\partial z^2} = \mu\sigma i\, h(z)(-i\omega)\, e^{-i\omega t}$$

$$\frac{\partial^2 h(z)}{\partial z^2} = -i\omega\mu\sigma\, h(z)$$

Exactly the same equation found in lecture and book, thus same solution:

$$h(z) = e^{-z/\delta} e^{iz/\delta}, \quad \delta = \sqrt{\frac{2}{\mu\sigma\omega}}$$

$$H(z,t) = H_0\, e^{-z/\delta} e^{iz/\delta}\, \underset{\substack{\uparrow \\ e^{i\pi/2}}}{i}\, e^{-i\omega t}$$

$$= H_0\, e^{-z/\delta}\, e^{i\left(\frac{z}{\delta} - \omega t + \pi/2\right)}$$

or $H_0\, e^{-z/\delta}\, i\, e^{i(z/\delta - \omega t)}$

$$\mathrm{Re}\, H(z,t) = H_0 e^{-z/\delta} \underbrace{\mathrm{Re}\left[i\, e^{i(z/\delta - \omega t)}\right]}_{\begin{array}{l} -\sin(z/\delta - \omega t) \\ = \sin(\omega t - z/\delta) \end{array}}$$

which for $z = 0^+$ it is correct.

The discussion about the exponential decay and skin depth is the same as in the lecture.

The solution for $\vec{H}$ is then:

$$\boxed{\vec{H} = \hat{e}_y\, H_0\, e^{-z/\delta} \sin(\omega t - z/\delta).}$$

Now we need the electric field $\vec{E}$.

From $\nabla \times \vec{H} = \vec{J} = \sigma \vec{E}$, we simply say:

$$\nabla \times \vec{H} = \begin{vmatrix} \hat{e}_x & \hat{e}_y & \hat{e}_z \\ \nabla_x & \nabla_y & \nabla_z \\ 0 & H_y & 0 \end{vmatrix} = -\hat{e}_x \nabla_z H_y + \hat{e}_z \underbrace{\nabla_x H_y}_{=0}$$

$$= \sigma\left(E_x \hat{e}_x + E_y \hat{e}_y + E_z \hat{e}_z\right)$$

$$H_y = H_0 e^{-z/\delta}\, i\, e^{i(z/\delta - \omega t)}$$

Then, $E_x = -\dfrac{\nabla_z H_y}{\sigma}$, $E_y = E_z = 0$

$$E_x = -\frac{1}{\sigma}\frac{d}{dz}\left[H_0 e^{-z/\delta}\, i\, e^{i(z/\delta - \omega t)}\right] =$$

$$= -\frac{i}{\sigma} H_0 e^{-i\omega t} \frac{d}{dz}\left[e^{-z/\delta} e^{iz/\delta}\right] =$$

$$= -\frac{i}{\sigma} H_0 e^{-i\omega t}\left[-\frac{1}{\delta} e^{-z/\delta} e^{iz/\delta} + e^{-z/\delta}\frac{i}{\delta} e^{iz/\delta}\right]$$

$$= \frac{-i}{\sigma} H_0 e^{-i\omega t} e^{-z/\delta} e^{iz/\delta} \frac{(-1+i)}{\delta} = \frac{H_0}{\sigma\delta} e^{-i\omega t} e^{-z/\delta} e^{iz/\delta} \underbrace{(1+i)}_{e^{i\pi/4}\sqrt{2}}$$

$$= \frac{H_0 \sqrt{2}}{\sigma\delta} e^{-z/\delta} e^{-i\omega t} e^{iz/\delta} e^{i\pi/4}$$

$\sigma = \dfrac{2}{\mu\omega\delta^2}$

$$= \frac{\mu\omega\delta H_0}{\sqrt{2}} e^{-z/\delta} \underbrace{\mathrm{Re}\left[e^{(-i\omega t + iz/\delta + i\pi/4)}\right]}_{\cos\left(\frac{z}{\delta} - \omega t + \pi/4\right)}$$

real part

or $\mathrm{Re}\left[e^{-i\omega t} e^{iz/\delta} \frac{(1+i)}{\sqrt{2}}\right] = \mathrm{Re}\, e^{-i\omega t} e^{iz/\delta}\, i \frac{(1-i)}{\sqrt{2}}$

$$= \mathrm{Re}\left[i\left[e^{i(\frac{z}{\delta} - \omega t)} \underbrace{e^{-i\pi/4}}_{\frac{1-i}{\sqrt{2}}}\right]\right]$$

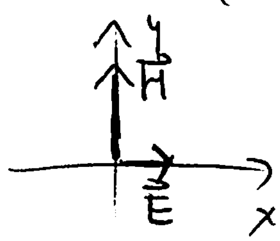
$$= -\sin\left(\frac{z}{\delta} - \omega t - \pi/4\right)$$

$$= +\sin\left(\omega t - \frac{z}{\delta} + \pi/4\right)$$

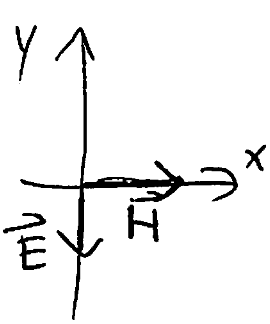
Final answer:

$$\boxed{\begin{aligned} \vec{H} &= \hat{e}_y H_0 e^{-z/\delta} \sin(\omega t - z/\delta) \\ \vec{E} &= \hat{e}_x \frac{\mu\omega\delta}{\sqrt{2}} H_0 e^{-z/\delta} \sin\left(\omega t - \frac{z}{\delta} + \pi/4\right) \end{aligned}}$$

If $z = 0^+$ and $\omega t = \pi/2$, then $\vec{H}$ points along the positive y-direction, while $\vec{E}$ points along x with magnitude $\sin(\pi/2 + \pi/4)$ which is still positive



In the case solved in class, for $\omega t = 0$ and $z = 0^+$ we get $\vec{H}$ along the x direction (positive) and $\vec{E}$ is along the y direction with a $\cos(3\pi/4)$ which is negative.



I.e. the orientations are the same as it has to be.