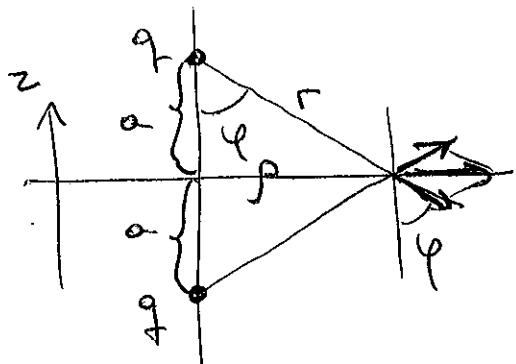


For the case of two charges of equal sign, the electric field at the plane in the middle between the charges points only along that plane:



$$|\vec{E} \text{ of each charge}| = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2}$$

thus, the magnitude of the field in the plane, double:

$$|\vec{E}_{\text{total in the plane}}| = \frac{2q}{4\pi\epsilon_0} \cdot \frac{p}{(a^2+p^2)^{1/2}}$$

$$\text{This is } \sqrt{E_x^2 + E_y^2} = |\vec{E}_{\text{total}}| \cdot \sin\varphi$$

In the formulas for the force in (6.122) we need to calculate

$$\oint_S \sum_{\beta} T_{\alpha\beta} n_{\beta} d\alpha$$

plane

The vector  $n_{\beta} = (0, 0, 1)$  implies

$$\oint_S \sum_{\beta} T_{\alpha\beta} n_{\beta} d\alpha = \oint_S T_{\alpha z} d\alpha$$

But  $T_{xz} = \epsilon_0 E_x E_z = 0$  since  $E_z = 0$  by symmetry at the plane  
 and  $T_{yz} = 0$  for the same reason.

Then only  $\oint T_{zz} da$  can be relevant.  
 S

$$T_{zz} = \epsilon_0 \left[ E_z E_z - \frac{1}{2} (E_z E_z + E_x E_x + E_y E_y) \right]$$

$$= -\frac{\epsilon_0}{2} \underbrace{(E_x^2 + E_y^2)}$$

in plane. This quantity was already calculated!

$$T_{zz} = -\frac{\epsilon_0}{2} \underbrace{|E_{\text{total}}|^2}_{\text{previous page}} = -\frac{\epsilon_0}{2} \cdot \left[ \frac{2q}{4\pi\epsilon_0(a^2 + j^2)} \cdot \frac{1}{(a^2 + j^2)^{1/2}} \right]^2$$

The full integral will be:

$$\oint_S T_{zz} da = \int_0^{2\pi} d\phi \int_0^\infty j dp \left(-\frac{\epsilon_0}{2}\right) \frac{4q^2}{16\pi^2\epsilon_0^2} \frac{j^2}{(a^2 + j^2)^3}$$

$$= -\frac{\epsilon_0}{2} \cdot \frac{4q^2}{16\pi^2\epsilon_0^2} \cdot 2\pi \int_0^\infty \frac{j^3 dp}{(a^2 + j^2)^3} =$$

$$= -\frac{\epsilon_0}{2} \cdot \frac{4q^2 \cdot 2\pi}{16\pi^2\epsilon_0^2} \cdot \frac{1}{4a^2}$$

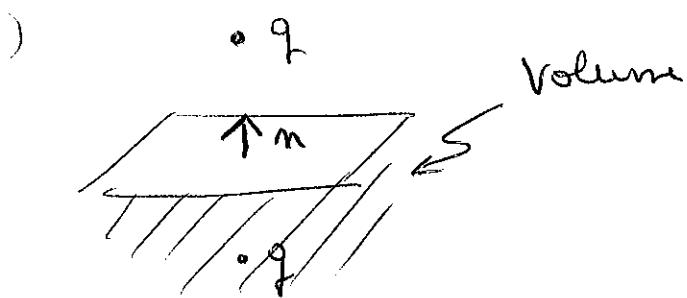
evaluated elsewhere  
in this solution

The magnitude is precisely what we expect of two pointlike charges separated by a distance  $(a)$  !

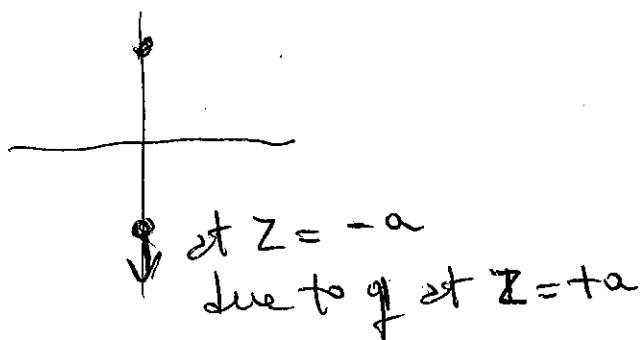
How about the sign?

From page 261 of Jackson, bottom, it says the integrand of 6.122 is "the force per unit area transmitted across the surface  $S$  and acting on the combined system of particles and fields inside  $V$ "

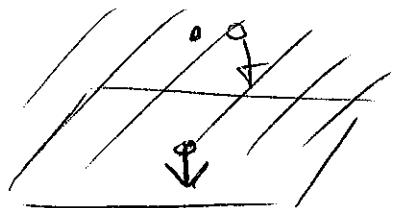
Since we chose  $\mathbf{n}_\beta = (0, 0, 1)$  i.e. pointing up, the volume chosen is the lower region  $z < 0$ .



The force we calculated is the force on  $q$  at  $z = -a$  (inside  $V$ ) caused by  $q$  at  $z = +a$  (outside  $V$ ). Then, it has to be a negative force, and the result we found is correct including the sign.



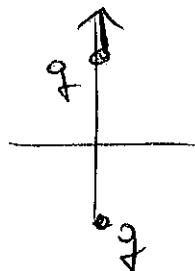
If we would have selected  $M_P = (0, 0, -1)$   
then it would have been:



$\bullet q$

The "force" would have been positive  $+\frac{1}{4\pi\epsilon_0} \cdot \frac{q^2}{(2a)^2}$

but this force would have been the force  
acting on  $q$  ( $z=a$ ) due to charge  $q$  at  $z=-a$ .



which is correct.

$$\int_0^\infty \frac{p^3 dp}{(a^2 + p^2)^3} = \int_a^\infty \frac{(u^2 - a^2) u du}{u^6} =$$

$$a^2 + p^2 = u^2$$

$$2p dp = 2u du$$

$$p^3 dp = p^2 \cdot p dp = (u^2 - a^2) u du$$

$$= \int_a^\infty \frac{du}{u^3} - a^2 \int_a^\infty \frac{du}{u^5} = \left( -\frac{1}{2u^2} \right) \Big|_a^\infty - a^2 \left( -\frac{1}{4u^4} \right) \Big|_a^\infty$$

$$= \frac{1}{2a^2} - a^2 \cdot \frac{1}{4a^4} = \frac{1}{2a^2} - \frac{1}{4a^2} = \frac{1}{4a^2}.$$