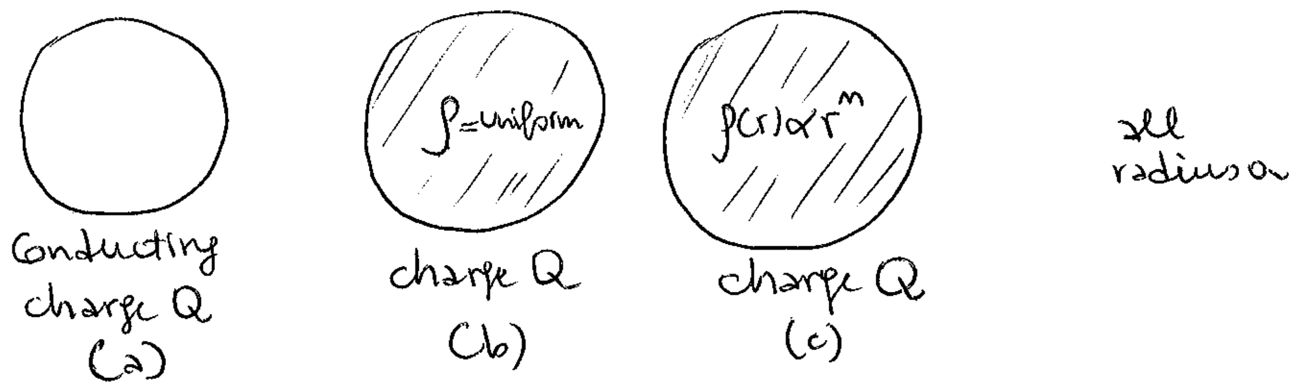
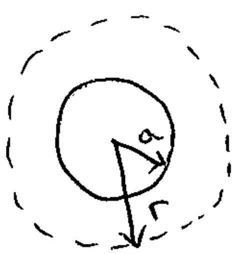
Problem 1.4

Three types of spheres:



(a) $\boxed{E_{inside} = 0}$ because it is a conducting sphere and a conducting sphere has $\vec{E} = 0$ when in equilibrium.

For Eoutside, we use Eq (1.9) employing a spherical surface of radius $r > a$.



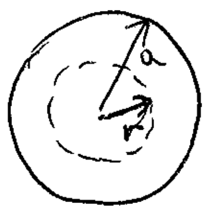
$$\oint_S \underbrace{\vec{E} \cdot \vec{n}}_{\substack{\vec{E} \text{ is radial} \\ \text{and constant} \\ \text{on the sphere} \\ \text{we are using.}}} da = E(r) \underbrace{4\pi r^2}_{\oint_S da} = Q/\epsilon_0$$

$$\boxed{\underset{\text{outside}}{E(r)} = \frac{Q}{4\pi r^2 \epsilon_0}}$$

(b) Since Q is given, then the density of charge assumed uniform must be

$$\rho = \frac{Q}{\text{Volume}} = \frac{Q}{\frac{4\pi}{3} a^3}$$

For E_{inside} we select a sphere of radius $r < a$ and apply Gauss' theorem:



$$E_{inside}(r) \cdot 4\pi r^2 = \frac{Q_{inside}}{\epsilon_0} = \frac{\rho \cdot \frac{4\pi r^3}{3}}{\epsilon_0}$$

$$E_{inside} = \frac{Q}{\frac{4\pi}{3} a^3} \, \frac{\frac{4\pi r^3}{3}}{4\pi r^2 \epsilon_0} = \boxed{\frac{Q r}{4\pi \epsilon_0 a^3}}$$

$E_{outside}$ is exactly the same as in (a) since the Q that appears in (1.9) can be at the surface or uniformly spread or whatever. Outside what matters is the total charge only.

(c) Again $E_{outside}$ same as in (a). Inside first we have to find the normalization of $\rho(r)$

$$Q = \int \rho(r)\, dV = A \underbrace{\int_0^a \rho^n \rho^2 \, d\rho}_{\left. \frac{\rho^{n+2+1}}{n+3} \right|_0^a} \underbrace{4\pi}_{\text{From solid angle}} ; \qquad \boxed{A = \frac{Q\,(n+3)}{4\pi \, a^{n+3}}}$$

Use Gauss theorem again:

$$E(r)\, 4\pi r^2 = \frac{Q_{inside}}{\epsilon_0} = \frac{1}{\epsilon_0}\int_0^r A\rho^n \rho^2 d\rho \; 4\pi$$

$$= \frac{4\pi}{\epsilon_0} A \left.\frac{\rho^{n+3}}{n+3}\right|_0^r = \frac{4\pi A}{\epsilon_0}\frac{r^{n+3}}{n+3}$$

$$E(r) = \frac{1}{4\pi r^2}\frac{\cancel{4\pi}}{\epsilon_0}\frac{Q\,\cancel{(n+3)}}{\cancel{4\pi}\, a^{n+3}}\frac{r^{n+3}}{\cancel{(n+3)}}$$

$$= \boxed{\frac{Q\, r^{n+1}}{4\pi\epsilon_0\, a^{n+3}}}$$

(d) Sketches: