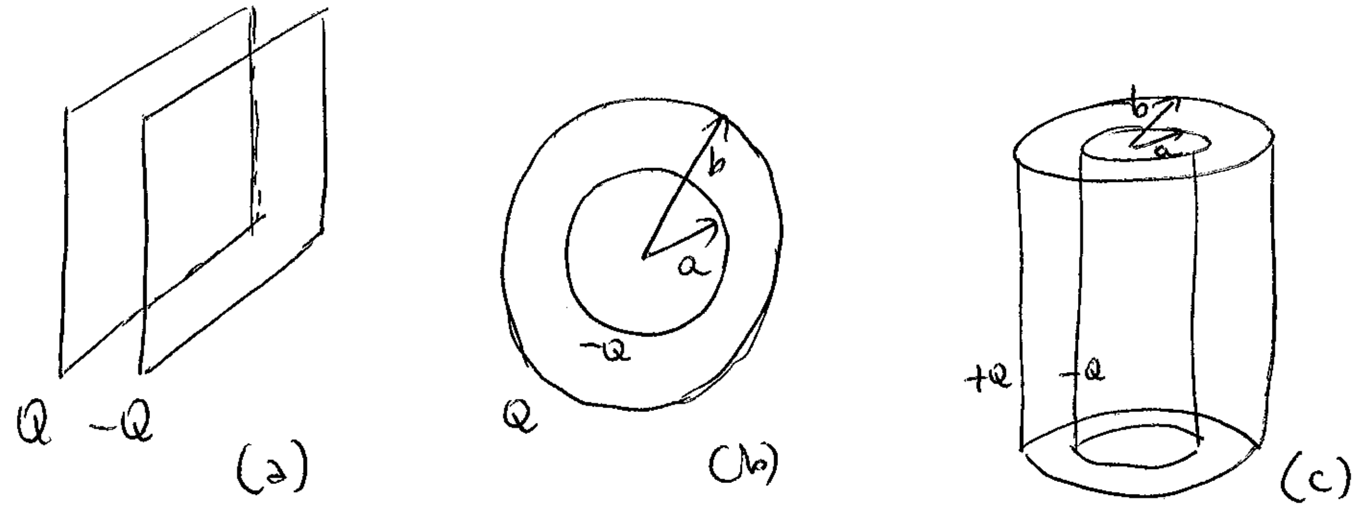
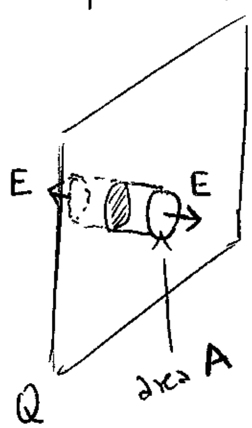
Problem 1.6 (only (a), (b), (c))



(a) Let us use Gauss's law with an imaginary cylinder, as sketch below, for one of the plates.

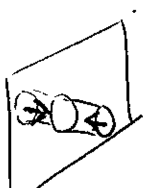


Only the top and bottom of the cylinder contributes because we know that for a large charged plane $\vec{E}$ is perpendicular to the plane.

$$\oint_S \vec{E} \cdot \hat{n}\, da = \frac{q_{inside}}{\epsilon_0}$$

$$\underset{\text{top+bottom}}{2}EA = \frac{1}{\epsilon_0}\underbrace{\sigma A}_{q_{inside}} \rightarrow E = \frac{\sigma}{2\epsilon_0} \text{ due to one plane}$$

Doing the same calculation for the other plane we also get $E = \frac{\sigma}{2\epsilon_0}$ but in the ~~opposite~~ same direction as the one before. So the combination gives:



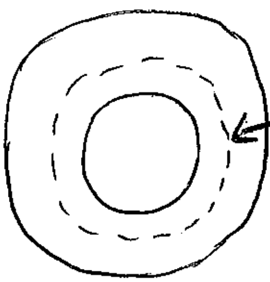
$$\boxed{E_{total} = \frac{\sigma}{\epsilon_0}}$$ Uniform

$$C = \frac{Q}{\underbrace{\Delta\Phi}_{E \cdot d}} = \frac{\sigma A^{total}}{d \cdot \frac{\sigma}{\epsilon_0}}$$

~~Capacitance~~
capacity

$$\boxed{C = \frac{\epsilon_0 A^{total}}{d}}$$

(b)

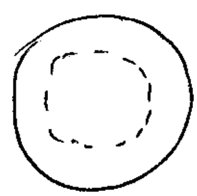


Let us use this imaginary surface to get the electric field of the inner sphere.

$$\oint_S \vec{E} \cdot \vec{n}\, da = \frac{q_{inside}}{\epsilon_0}$$

$$E_r 4\pi r^2 = \frac{Q}{\epsilon_0}, \quad E_r = \frac{Q}{4\pi\epsilon_0 r^2}$$

With respect to the larger sphere,



the same construction gives zero field because inside the larger sphere there is no charge caused by that larger sphere.

Then, total $\boxed{E = \frac{Q}{4\pi\epsilon_0 r^2}}$

To get $\Delta\Phi$ we simply recall $\vec{E} = -\nabla\Phi$

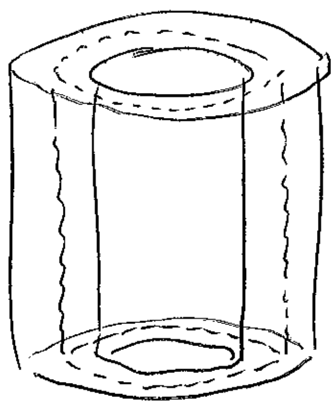
$$E_r = -\frac{\partial}{\partial r}\Phi$$

or $|\Delta\Phi| = \left| -\int_a^b E_r\, dr \right| =$

$$|\Delta\Phi| = \left|\frac{Q}{4\pi\epsilon_0}\underbrace{\int_a^b \frac{dr}{r^2}}_{-\frac{1}{r}\Big|_a^b = -\frac{1}{b}+\frac{1}{a}}\right| = \frac{Q}{4\pi\epsilon_0}\frac{(b-a)}{ab}$$

$$C = |\Delta\Phi|^{-1} Q = \boxed{\frac{4\pi\epsilon_0 ab}{(b-a)}}$$

(c) Conducting cylinders:



Again, use Gauss's law for the imaginary surface shown. We know, by symmetry, that the electric field will be radial.

$$\oint_S \vec{E}\cdot\vec{n}\,da = E(r)\underbrace{L\,2\pi r}_{\text{Area of cylinder}} = \frac{Q}{\epsilon_0}$$

$$\boxed{E(r) = \frac{Q}{2\pi\epsilon_0 r L}}$$

note that I am not paying attention to whether we have a $+Q$ or $-Q$ in the inner cylinder. The result is independent of that.

To get $\Delta\Phi$ we do:

$$|\Delta\Phi| = \int_a^b E(r)\,dr = \frac{Q}{2\pi\epsilon_0 L}\int_a^b \frac{dr}{r} = \frac{Q}{2\pi\epsilon_0 L}\ln(b/a)$$

$$\boxed{C = \frac{Q}{|\Delta\Phi|} = \frac{2\pi\epsilon_0 L}{\ln(b/a)}}$$