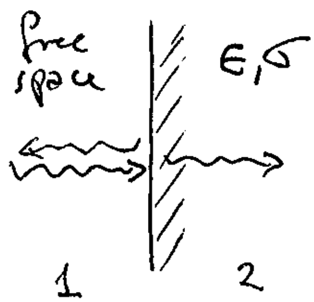
Problem 7.4 (a)

In this problem, we must use (7.57) where an $\epsilon(\omega)$ is defined as

$$\epsilon(\omega) = \epsilon + i\frac{\sigma}{\omega}$$



As done before in previous problems, we can setup the boundary conditions for the tangencial components of $\vec{E}$ and $\vec{H}$ and request continuity. Moreover, we can assume that μ of the medium is $\approx \mu_0$.

Then:

$$\left.\begin{aligned} E_1^i + E_1^r &= E_2^t \\ E_1^i - E_1^r &= \frac{k_2}{k_1} E_2^t \end{aligned}\right\}$$

From solution of Problem 7.2 (b) dropping the reflected wave in medium 2.

$$E_1^i = \frac{1}{2}\left(1 + \frac{k_2}{k_1}\right) E_2^t$$

$$E_1^r = \frac{1}{2}\left(1 - \frac{k_2}{k_1}\right) E_2^t$$

$$\frac{E_1^r}{E_1^i} = \frac{1 - \frac{k_2}{k_1}}{1 + \frac{k_2}{k_1}}$$

From (7.36):

$$\frac{k_2}{k_1} \underset{\uparrow}{=} \sqrt{\frac{\epsilon_2}{\epsilon_1}} = \sqrt{\frac{\epsilon + \frac{i\sigma}{\omega}}{\epsilon_0}} = \sqrt{\frac{\epsilon}{\epsilon_0} + i\frac{\sigma}{\omega\epsilon_0}}\,.$$

$\mu_1 = \mu_2$ for simplicity

Then:

$$\frac{E_1^r}{E_1^i} = \frac{1 - \sqrt{\frac{\epsilon}{\epsilon_0} + \frac{i\sigma}{\omega\epsilon_0}}}{1 + \sqrt{\frac{\epsilon}{\epsilon_0} + \frac{i\sigma}{\omega\epsilon_0}}} = A e^{i\varphi}$$

we need to find more explicitly A and φ.

Calling $n = \sqrt{\frac{\epsilon}{\epsilon_0} + \frac{i\sigma}{\omega\epsilon_0}}$, then

$$\boxed{[A]^2 = Ae^{i\varphi} A e^{-i\varphi} = \left(\frac{1-n}{1+n}\right)\left(\frac{1-n^*}{1+n^*}\right) = \frac{1+|n|^2 - 2\,\mathrm{Re}\,n}{1+|n|^2 + 2\,\mathrm{Re}\,n}}$$

where $\boxed{|n|^2 = n . n^* = \sqrt{\frac{\epsilon}{\epsilon_0} + \frac{i\sigma}{\omega\epsilon_0}}\sqrt{\frac{\epsilon}{\epsilon_0} - \frac{i\sigma}{\omega\epsilon_0}} = \sqrt{\left(\frac{\epsilon}{\epsilon_0}\right)^2 + \frac{\sigma^2}{\omega^2\epsilon_0^2}}}$

$$\mathrm{Re}\,n = \frac{n+n^*}{2} = \frac{1}{2}\left[\underbrace{\sqrt{\frac{\epsilon}{\epsilon_0} + \frac{i\sigma}{\omega\epsilon_0}}}_{|n|e^{i\frac{\alpha}{2}}} + \underbrace{\sqrt{\frac{\epsilon}{\epsilon_0} - \frac{i\sigma}{\omega\epsilon_0}}}_{|n|e^{-i\alpha/2}}\right] = |n|\cos\left(\frac{\alpha}{2}\right)$$

where $\boxed{\alpha = \tan^{-1}\left(\frac{\sigma}{\omega\epsilon}\right)}$

The overall phase φ is:

$$\varphi = \tan^{-1}\left\{\frac{\operatorname{Im}(E_i^r/E_i^i)}{\operatorname{Re}(E_i^r/E_i^i)}\right\}$$

$$\frac{E_i^r}{E_i^i} = \frac{1-n}{1+n} = \underbrace{\sqrt{\left(\frac{1-n}{1+n}\right)\left(\frac{1-n^*}{1+n^*}\right)}}_{\text{already found}}\, e^{i\varphi}$$

$$\frac{1-n}{1+n}\cdot\frac{1+n^*}{1+n^*} = \frac{1-|n|^2-\overbrace{(n-n^*)}^{\operatorname{Im}(n)2i}}{1+|n|^2+2\operatorname{Re} n}$$

$$\tan\varphi = \frac{-2\operatorname{Im}(n)\big/\cancel{(1+|n|^2+2\operatorname{Re} n)}}{(1-|n|^2)\big/\cancel{(1+|n|^2+2\operatorname{Re} n)}} = \frac{-2\operatorname{Im}(n)}{1-|n|^2}.$$

$$\operatorname{Im}(n) = |n|\sin\left(\frac{\alpha}{2}\right)$$

$$\boxed{\begin{aligned}&\varphi = \tan^{-1}\left(\frac{-2|n|\sin\left(\frac{\alpha}{2}\right)}{1-|n|^2}\right)\\ &\text{with } \alpha = \tan^{-1}\left(\frac{\sigma}{\omega\epsilon}\right)\\ &\qquad\text{and } |n|^2 = \sqrt{\left(\frac{\epsilon}{\epsilon_0}\right)^2 + \frac{\sigma^2}{\omega^2\epsilon_0^2}}\end{aligned}}$$