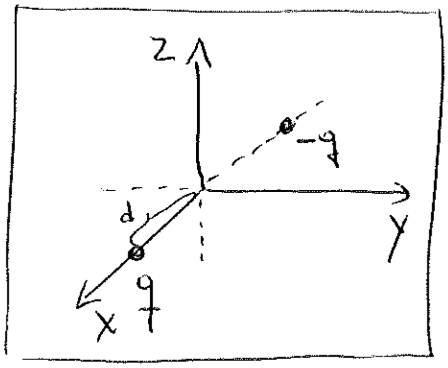
Problem 2.1 (only (a), (b), (c), (d). In (a), just sketch by hand)



$$\Phi = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{\sqrt{\underbrace{(x-d)^2}_{x^2+d^2-2xd}+y^2+z^2}} - \underbrace{\frac{1}{\sqrt{(x+d)^2+y^2+z^2}}}_{\text{opposite charge}}\right]$$

$\vec{x}' = (d,0,0)$

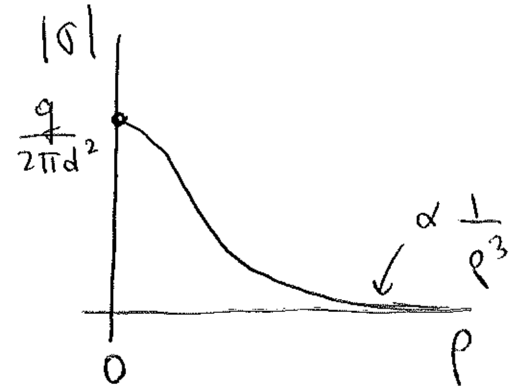
$\vec{x}''_{\text{image}} = (-d,0,0)$

(a) Surface charge density. See (2.5)

$$\sigma = -\epsilon_0 \left.\frac{\partial \Phi}{\partial x}\right|_{\substack{x=0\\ \text{plane}\\ (\text{i.e. } x=0)}} = \frac{-\epsilon_0 q}{4\pi\epsilon_0}\left[\left(-\frac{1}{2}\right)\frac{(2x-2d)}{((x-d)^2+y^2+z^2)^{3/2}} - \left(-\frac{1}{2}\right)\frac{(2x+2d)}{((x+d)^2+y^2+z^2)^{3/2}}\right]\Bigg|_{\substack{x=0\\ \text{plane}}} =$$

$$= \frac{-\epsilon_0 q}{4\pi\epsilon_0}\left[\frac{d}{(d^2+y^2+z^2)^{3/2}} + \frac{d}{(d^2+y^2+z^2)^{3/2}}\right]$$

$$= -\frac{qd}{2\pi}\cdot\frac{1}{(d^2+\underbrace{y^2+z^2}_{\rho^2})^{3/2}}$$

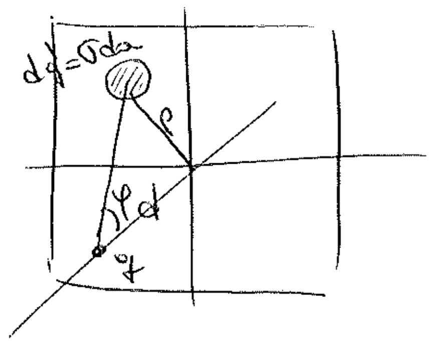


(b) The force on the charge q is caused by the image charge $q' = -q$

$$F = \frac{q q'}{4\pi\epsilon_0 |\text{distance}|^2} = \boxed{\frac{-q^2}{4\pi\epsilon_0 (2d)^2}}$$

Since we have the potential Φ from (a), we could have calculated the electric field $-\frac{\partial \Phi}{\partial x}$ and then multiply by q, but it has to be the Φ without the own potential of q, just the image.

(c) See before the discussion on page 60.



We only focus on the x component since the rest cancels by symmetry

$$dF_x = \frac{1}{4\pi\epsilon_0} \frac{q\,(\sigma\, da)}{\underbrace{\left(\sqrt{d^2+\rho^2}\right)^2}_{=|\vec{x}|^2}} \underbrace{\cos\varphi}_{\text{to project in } x} \qquad \cos\varphi = \frac{d}{\sqrt{d^2+\rho^2}}$$

$$dF_x = \frac{q\,\sigma\, da}{4\pi\epsilon_0 (\rho^2+d^2)} \frac{d}{(\rho^2+d^2)^{1/2}}$$

we only care about the magnitude

$$= \frac{q}{4\pi\epsilon_0} \left[\frac{+q d}{2\pi (d^2+\rho^2)^{3/2}}\right] \frac{(\rho\, d\rho\, d\phi)\; d}{(\rho^2+d^2)^{3/2}}$$

$$F_x = \frac{+q^2 d^2}{4\pi\epsilon_0} \int_0^\infty \frac{d\rho\; \rho}{(\rho^2+d^2)^{3}} = \frac{+q^2 d^2}{4\pi\epsilon_0} \left[-\frac{1}{4}(\rho^2+d^2)^{-2}\right]_0^\infty$$

$\int_0^{2\pi} d\phi = 2\pi$

$$= \frac{q^2}{4\pi\epsilon_0 d^2 4} = \boxed{\frac{q^2}{4\pi\epsilon_0 (2d)^2}}$$

This is the same result as in (b).

(d)

$$W = \int_d^\infty \vec{F} \cdot d\vec{x} = \int_d^\infty \frac{q^2}{4\pi\epsilon_0 (2x)^2}\, dx = \frac{q^2}{4\pi\epsilon_0 4} \left(-x^{-1}\right)\Big|_d^\infty$$

locate q at an arbitrary position x along the x-axis

$$= \boxed{\frac{q^2}{16\pi\epsilon_0 d}}$$