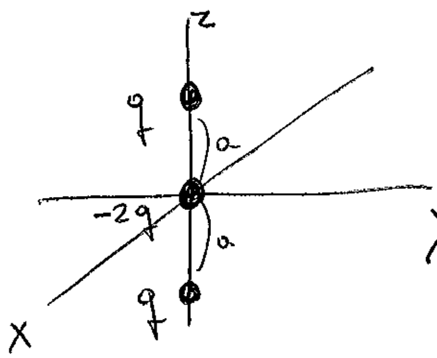
# Problem 4.1, part (b) only

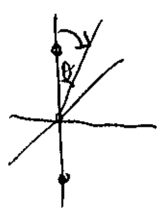



We will use the formula Eq. (4.3):

$$q_{lm} = \int Y_{lm}^{*}(\theta', \phi') r'^{l} \rho(\vec{x}') d^3x'$$

$$\rho(\vec{x}') = \underbrace{q\,\delta(\vec{x}' - a\hat{k})}_{\text{i.e. } r' = a} + \underbrace{q\,\delta(\vec{x}' + a\hat{k})}_{\text{i.e. } r' = a} - \underbrace{2q\,\delta(\vec{x}')}_{\text{i.e. } r' = 0}$$

$$q_{lm} = \left[ Y_{lm}^{*}(0,0)\, a^l q + Y_{lm}^{*}(\pi, 0)\, a^l q + 0 \right] \quad \text{for } l > 0$$




For $l = 0$, $q_{00} = 0$
because $\int \rho(\vec{x}') d^3x' = 0$.

Also since there is azimuthal symmetry, we cannot have a dependence on $\phi$, thus $\underline{\underline{m = 0}}$.

$$q_{l0} = Y_{l0}^{*}(0,0)\, a^l q + Y_{l0}^{*}(\pi, 0)\, a^l q$$

Let us now use Eq.(3.53):

$$Y_{l0}(0,0) = \sqrt{\frac{2l+1}{4\pi}}\, \underbrace{P_l^0(\cos\overset{\theta=0}{\theta})}_{\equiv P_l(\cos\theta)}$$

(see (3.49))

$$Y_{l0}(\pi,0) = \sqrt{\frac{2l+1}{4\pi}}\, P_l(\underbrace{\cos\pi}_{-1})$$

So we need $P_l(1)$ and $P_l(-1)$

From table of Legendre polynomials (3.15) we see:
$$P_l(\pm 1) = \begin{cases} 1 & \text{for all } l\text{'s if } x(\equiv\cos\theta) = 1 \ (\theta=0) \\ (-1)^l & \text{" " } x = -1 \ (\theta=\pi) \end{cases}$$

Then, $q_{l0} = q a^l \sqrt{\frac{2l+1}{4\pi}} \left[1 + (-1)^l\right]$

which cancels for $l$ odd.

$$\underset{l\,\text{even}}{q_{l0}} = q a^l \sqrt{\frac{2l+1}{4\pi}}\, 2 = \boxed{q a^l \sqrt{\frac{2l+1}{\pi}}}$$

Explicitly: $l=2$ gives
$$\boxed{\begin{array}{l} q_{20} = q a^2 \sqrt{\frac{5}{\pi}} \\ q_{40} = q a^4 \sqrt{\frac{9}{\pi}} \end{array}}$$

So this distribution of charge has both the monopole and dipole moments equal to 0.