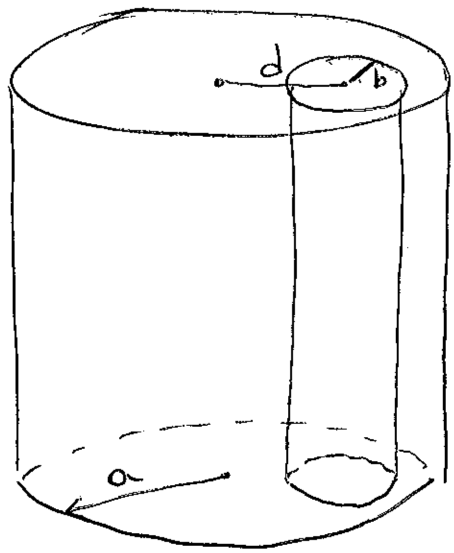
Problem 5.6



The best way to solve this problem is to superpose the field created by the cylinder "a" assumed solid with current $\vec{J}$ and the cylinder "b" also assumed solid with current $-\vec{J}$.

We will use Ampere's law.

$$\oint_C \vec{B}\cdot\vec{dl} = \mu_0 \times \begin{pmatrix}\text{Current}\\ \text{enclosed}\\ \text{by } C\end{pmatrix}$$

$$\oint_C \vec{B}\cdot\vec{dl} = \int_A \underbrace{(\nabla\times\vec{B})}_{\mu_0 \vec{J} \text{ (see (5.26))}}\cdot\vec{da} = \mu_0 \int_A \vec{J}\cdot\vec{da} = \mu_0 I$$

Stoke's theorem

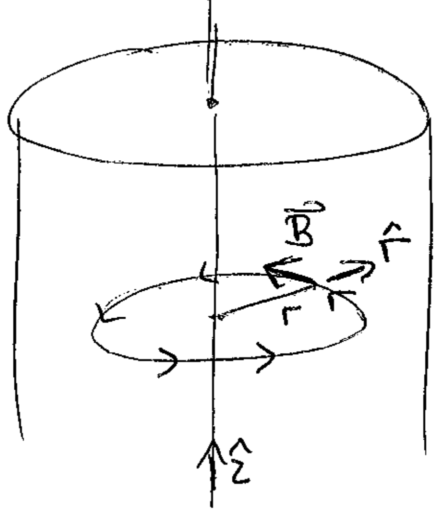
see (5.25)

For a circular loop centered at axis of cylinder "a" of radius r:

$|\vec{J}|$

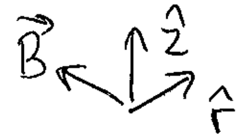
$$B\, 2\pi r = \mu_0 J \pi r^2$$

$$B = \frac{\mu_0 J r}{2}$$



$$\vec B = \frac{\mu_0 J r}{2}\left(\hat z \times \hat r\right)$$

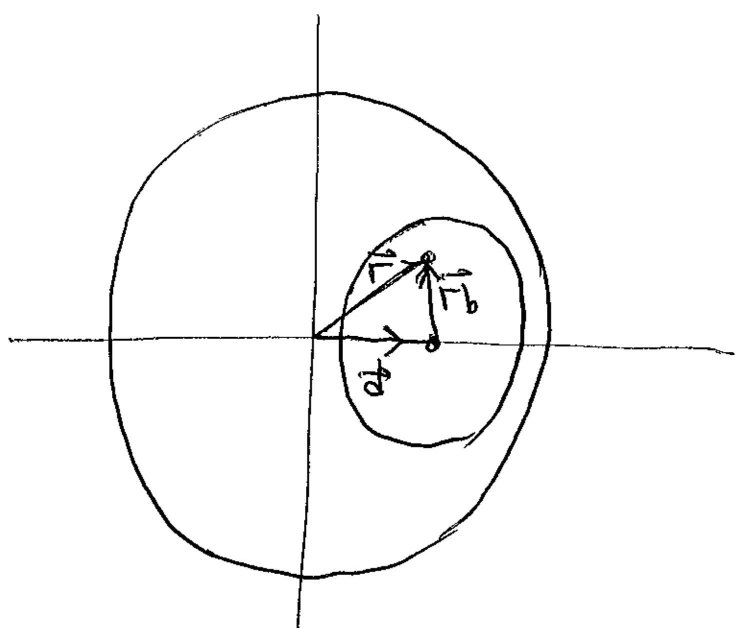
Unit vectors



$$= \frac{\mu_0 J}{2}\left(\hat z \times \vec r\right)$$

$\vec r = r\hat r$

For the <u>inside</u> of cylinder "b" we repeat the procedure or directly take the results of cylinder "a" and change $\vec r$ into $\vec r - \vec d$ as the geometry suggests:



$$\vec r = \vec r_b + \vec d$$

$$\vec r_b = \vec r - \vec d$$

$\vec r_b$ is the vector that matters in cylinder "b"

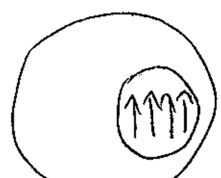
Also remember that in "b" the current goes the other way. Thus:

$$\underset{\substack{\text{inside}\\ \text{cylinder "b"}\\ \text{(not outside)}}}{\vec{B}} = \underbrace{\frac{\mu_0 J}{2}(\hat{z}\times\vec{r})}_{\text{From "solid a"}} \underset{\substack{\text{different}\\ \text{sense of } \vec{J}}}{-} \frac{\mu_0 J}{2}\left[\hat{z}\times(\vec{r}-\vec{d})\right]$$

$$= \boxed{\frac{\mu_0 J}{2}(\hat{z}\times\vec{d})}$$

If $\vec{d} = d\vec{i}$ as in the previous figure,
then $\hat{z}\times\vec{i} = \vec{j}$ (unit vector along y axis)

$$\vec{B}_{\substack{\text{inside}\\ \text{"hole"}}} = \frac{\mu_0 J}{2} d\vec{j}$$



It is remarkable that inside the hole the magnetic field is constant (i.e. uniform)

If you wish to express this in terms of the total current $I = J(\pi a^2 - \pi b^2)$, it is

$$\left|\vec{B}_{\text{inside}}\right| = \frac{\mu_0 I}{2\pi(a^2-b^2)} d|\vec{j}| = \frac{\mu_0 I d}{2\pi(a^2-b^2)}$$