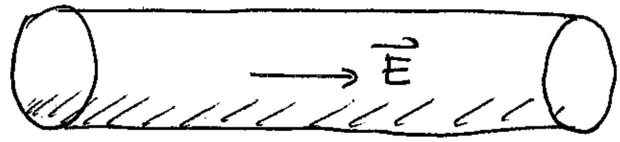
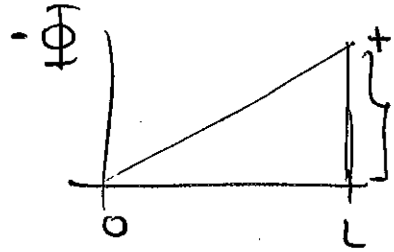
Griffiths
page 324, Ex. 15

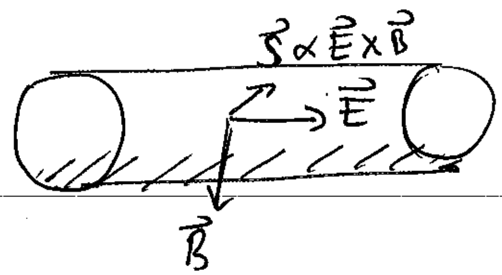


$\vec{E}$ assumed uniform
is of value $\frac{V}{L}$
because we know a $\Phi = -Ex$
causes a uniform E since $-\frac{d\Phi}{dx} = E$



we call this V, thus $\boxed{E = \frac{V}{L}}$
(the pot. diff. between the ends)

The magnetic field is $|B| = \frac{\mu_0 I}{2\pi R}$ from elementary considerations.
(at the surface)
(page 209 Griffiths)



$$|\vec{S}| = \frac{1}{\mu_0}|\vec{E} \times \vec{B}| = \frac{1}{\mu_0}|\vec{E}||\vec{B}| = \frac{1}{\cancel{\mu_0}} \cdot \frac{V}{L} \cdot \frac{\cancel{\mu_0} I}{2\pi R} = \frac{VI}{2\pi R L}$$

From (6.111) Jackson:

($\vec{S}$ points radially inwards)

$$-\oint \vec{S} \cdot d\vec{a} = +|\vec{S}| \, \text{area} = \frac{VI}{2\pi R L} 2\pi R L = \boxed{VI}$$

Surface of wire

$|\vec{S}|$ is constant at the surface. $\vec{S}$ and $\vec{n}$ point in opposite directions

If there is a resistance, this is spent as heat.

Energy per unit time passing through the surface of the wire.

Note that $-\oint \vec{S}\cdot d\vec{a}$ is overall positive, i.e. there is energy coming into the wire.

This energy does not go into the electromagnetic field since inside $\vec{E}$ and $\vec{B}$ are constant, then $\frac{d}{dt}\int (E^2 + c^2B^2)\, d^3x = 0$.

Thus, this increase in energy is taken by the electrons and atoms inside the wire, and this is in the form of heat that increases the temperature.

So the term $\int \vec{E}\cdot\vec{J}\, d^3x$ must be the important one:

$$\int \vec{E}\cdot\vec{J}\, d^3x \underset{\substack{\uparrow \\ \vec{E} \text{ is constant} \\ \vec{E},\vec{J} \text{ in same direction}}}{=} E\int J\, d^3x = E \underset{\text{(cross section)}}{\frac{I}{\text{Area}}} \cdot \text{Area} \cdot \text{Length} = EIL$$

$$= \frac{V}{L}\cdot I \cdot L$$

$$= V \cdot I$$

as expected.

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