(a)

Problem 5 of Chapter 6

From (6.117):

$$\vec{P}_{field} = \mu_0 \epsilon_0 \int_V d^3x \left(\vec{E} \times \vec{H} \right)$$

From (6.9): $\vec{E} = -\nabla\Phi - \frac{\partial \vec{A}}{\partial t}$ in general

but this problem is time independent, thus $\frac{\partial \vec{A}}{\partial t} = 0$.

$$\vec{P}_{field} = +\mu_0\epsilon_0 \int_V d^3x \left((-\nabla\Phi) \times \vec{H} \right)$$

Moreover: $\nabla \times (\Phi \vec{H}) = \nabla\Phi \times \vec{H} + \Phi(\nabla \times \vec{H})$

or $-\nabla\Phi \times \vec{H} = \Phi(\nabla \times \vec{H}) - \nabla \times (\Phi \vec{H})$

$$\vec{P}_{field} = \mu_0 \epsilon_0 \int_V d^3x \ \Phi(\nabla \times \vec{H}) - \mu_0\epsilon_0 \int_V d^3x \ \nabla \times (\Phi \vec{H})$$

From (6.62):

$$\nabla \times \vec{H} = \vec{J} - \frac{\partial \vec{D}}{\partial t} \underset{\text{if time independence assumed}}{=} \vec{J}$$

Then:

$$\boxed{\vec{P}_{field} \underset{\mu_0\epsilon_0 = \frac{1}{c^2}}{=} \frac{1}{c^2} \int_V d^3x \ \Phi \vec{J} - \frac{1}{c^2} \underbrace{\int_V d^3x \ \nabla \times (\Phi \vec{H})}_{\oint_S \hat{n} \times (\Phi \vec{H}) da}}$$

"da" contains a "R^2" where R is the radius of the sphere that contains the currents. Then, $|\Phi \vec{H}|$ has to decay faster than $1/R^2$ for this term to vanish as $R \to \infty$.

Note that in principle a multipole expansion of Φ and $\vec{H}$ gives particular powers of $1/r$ that can exist, although mathematically $1/R^{2+\epsilon}$ is enough.