Chapter 11

Special Theory of Relativity

We will be covering only a special subset of the many sections of this chapter. Our goal is to focus on the Maxwell equations, to show they are Lorentz invariant, and to study how $\vec{E}$ and $\vec{B}$ transform. A lot of the other sections are not relevant for us or assumed to be known by the students from previous classes.

We will talk about Galilean transf. (11.1) and Lorentz transf. (11.16). The Galilean transf. are:

$$\vec{x}' = \vec{x} - \vec{v}t$$
$$t' = t$$

between two systems of coordinates K and K' moving one respect of the other at speed $\vec{v}$.

It can be shown that the usual equations of the form $\vec{F} = m\vec{a}$ are invariant under Galilean transformations. More specifically,

$$m_i \frac{d\vec{v}_i'}{dt'} = -\nabla_i' \sum_j V_{ij}(|\vec{x}_i' - \vec{x}_j'|) \quad (11.2)$$

when transformed from K' to K has exactly the same form but without the primes i.e.

$$m_i \frac{d\vec{v}_i}{dt} = -\nabla_i \sum_j V_{ij}(|\vec{x}_i - \vec{x}_j|) \quad (11.3)$$

(this was shown in separate pages).

This is like "common sense" i.e. whether you are on Earth or in rocket in outer space or anywhere without acceleration, the formulas have to be the same. Note that this "common sense" statement does not say that Newton's law are right and the Galilean transf. are right. It says that for the correct laws (which later we will show are those of relativity) and for the correct transformation, the laws in K and K' have to be the same.

What about the Maxwell equations?

they are of the wave equation form.

It is shown in different pages that the equation

$$\left(\sum_i \frac{\partial^2}{\partial x_i'^2} - \frac{1}{c^2} \frac{\partial^2}{\partial t'^2} \right) \psi = 0$$

becomes in K

$$\left[\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \frac{2}{c^2} (\vec{v} \cdot \vec{\nabla}) \frac{\partial}{\partial t} - \frac{1}{c^2} (\vec{v} \cdot \vec{\nabla})(\vec{v} \cdot \vec{\nabla}) \right] \psi = 0$$

which is obviously <u>different</u>. Thus, the Maxwell's equations are <u>not</u> galilean invariant! What is going on? Note that this lack of invariance holds not only for Maxwell's equations but for any wave equation such as that of sound of waves in water. <u>However</u>, for sound or water waves we do <u>not</u> expect the equations to be the same because they involve a <u>medium</u> (air, water) <u>which is at rest only in one system of coordinates</u>. Thus, nobody expects the sound or waves eqs. to be galilean invariant. <u>But Max. Eqs. are different since there is no evidence whatsoever of an ether!</u> As far as we know $\vec{E}$ and $\vec{B}$ exists in totally empty space in the middle of galaxies. Thus, they should be galilean invariant!

So what are the options then ?

* Max. Eqs. are wrong and the "correct" ones are galilean invariant. But Max. Eqs. experimentally seem correct !
* Max. Eqs. are right and they are not galilean inv. because of the ether. But there is no ether !
* If Max. Eqs. are right, and there is no ether, they must be invariant under a "generalized galilean invariance" that we do not know (as of 1905). Newton's law should be invariant under the same generalized law. Thus, Newton's law needs to be generalized as well !!

Einstein followed the last path and found that the generalized galilean transf. are the <u>Lorentz transf.</u>
But the Maxwell equations involve the speed of light!
If the Max. equations in K and K' are to be the same then $c' = c$ i.e. <u>the speed of light has to be the same independent of the relative velocity of K and K'!</u> This is a totally radical not-intuitive statement that says that waves in E&M are totally different from waves in air and water!

Are the equations of classical mechanics invariant under a Galilean transf.?

Consider two particles interacting via a potential $V(|\vec{x}_1 - \vec{x}_2|)$. The eqs. of classical mechanics are:

$$m_1 \frac{d\vec{v}_1}{dt} = -\frac{d}{dx_1} V(|x_1 - x_2|)$$

$$m_2 \frac{d\vec{v}_2}{dt} = -\frac{d}{dx_2} V(|x_1 - x_2|)$$

let us actually assume 1D (space) for simplicity

Let us write this in the K' frame i.e. we simply add "primes" everywhere:

$$m_1 \frac{dv_1'}{dt'} = -\frac{d}{dx_1'} V(|x_1' - x_2'|)$$

$$m_2 \frac{dv_2'}{dt'} = -\frac{d}{dx_2'} V(|x_1' - x_2'|)$$

(Note that we assume $m_1 = m_1'$, $m_2 = m_2'$ which in relativity is <u>not</u> correct)

and see if we can go back to a system K keeping the same functional form. To start with, we know that $t' = t$, thus $\frac{d}{dt'} = \frac{d}{dt}$.

then, we say
$$x_1'(t) = x_1(t) - vt$$
$$x_2'(t) = x_2(t) - vt$$

which is (11.1). Note that x is a function of t as opposed to an independent variable as it happens in the wave equation where we have a function $\phi(x,t)$

$$\frac{dx_1'(t)}{dt} = \frac{dx_1(t)}{dt} - v \quad \text{i.e.} \quad v_1'(t) = v_1(t) - v$$

$$\frac{dx_2'(t)}{dt} = \frac{dx_2(t)}{dt} - v \quad \text{i.e.} \quad v_2'(t) = v_2(t) - v$$

Standard addition of velocities which is not true in relativity

$$\frac{dx_2'(t)}{dt} \equiv v_2'(t)$$

then $\frac{dv_1'(t)}{dt} = \frac{dv_1(t)}{dt}$

$$\frac{dv_2'(t)}{dt} = \frac{dv_2(t)}{dt}$$

As a consequence:

$$m_1 \frac{dv_1'(t')}{dt'} \underset{t'=t}{=} m_1 \frac{dv_1'(t)}{dt} = m_1 \frac{dv_1(t)}{dt}$$

$$m_2 \frac{dv_2'(t')}{dt'} = m_2 \frac{dv_2(t)}{dt}$$

Thus, the "ma" portion of the F=ma equation <u>is</u> galilean invariant!

Consider now the "F" side. To start with:

$$|x_1'(t') - x_2'(t')| \underset{t'=t}{=} |x_1'(t) - x_2'(t)| = |x_1(t) - vt - (x_2(t) - vt)|$$
$$= |x_1(t) - x_2(t)|$$

thus the potential V is invariant in form.

With regards to $\frac{d}{dx_1'}$, use the chain rule: $\frac{d}{dx_1'} = \frac{d}{dx_1} \underbrace{\frac{dx_1}{dx_1'}}_{1} = \frac{d}{dx_1}$

and the same for $\frac{d}{dx_2'}$. Thus, the "F" term is galilean invariant

Is the wave equation (11.4) galilean invariant?

The equation is

$$\left(\sum_{i=1}^{3}\frac{\partial^2}{\partial x_i'^2}-\frac{1}{c^2}\frac{\partial^2}{\partial t'^2}\right)\phi(\vec{x}',t')=0$$

Note that ψ is a <u>field</u> i.e. a function of $\vec{x}$ and t, both of them being independent of each other. ψ for instance could be an electric field $\vec{E}(\vec{x},t)$ or a potential $\phi(\vec{x},t)$ where we do not assume any $\vec{x}=\vec{x}(t)$ as we do for the movement of a pointlike particle.

In view of this independence between $\vec{x}$ and t consider the <u>chain rule</u> (just for the case of 1D for simplicity):

$$\frac{d}{dt'}\psi=\frac{dx}{dt'}\frac{d\psi}{dx}+\frac{dt}{dt'}\frac{d\psi}{dt}$$

But since $\begin{matrix}x'=x-vt\\ t'=t\end{matrix}$ then inverting $\begin{matrix}x=x'+vt'\\ t=t'\end{matrix}$

and $\frac{dt}{dt'}=1$, $\frac{dx}{dt'}=v$

and

$$\frac{d}{dt'}\psi=v\frac{d\psi}{dx}+\frac{d\psi}{dt}=\left(\frac{d}{dt}+v\frac{d}{dx}\right)\psi$$

Then, as operator:

$$\boxed{\frac{d}{dt'} = \frac{d}{dt} + v\frac{d}{dx}}$$ for a Galilean transf.

(or returning to 3D:

$$\frac{d}{dt'} = \frac{d}{dt} + \vec{v}\cdot\vec{\nabla}$$)

With regards to $\frac{d}{dx'}$ we do the same chain rule:

$$\frac{d\psi}{dx'} = \frac{dx}{dx'}\frac{d\psi}{dx} + \frac{dt}{dx'}\frac{d\psi}{dt}$$

But $\frac{dx}{dx'} = 1$ and $\frac{dt}{dx'} = 0$ since $\begin{matrix} x = x' + vt' \\ t = t' \end{matrix}$

Then $\frac{d\psi}{dx'} = 1 . \frac{d\psi}{dx} + 0\frac{d\psi}{dt}$ or

$$\boxed{\frac{d}{dx'} = \frac{d}{dx}}$$ for a Galilean transf.

Then, $$\frac{d^2}{dx'^2} - \frac{1}{c^2}\frac{d^2}{dt'^2} = \frac{d^2}{dx^2} - \frac{1}{c^2}\left(\frac{d}{dt} + v\frac{d}{dx}\right)\left(\frac{d}{dt} + v\frac{d}{dx}\right)$$

$$= \boxed{\frac{d^2}{dx^2} - \frac{1}{c^2}\frac{d^2}{dt^2} - \frac{2v}{c^2}\frac{d}{dx}\frac{d}{dt} - \frac{v^2}{c^2}\frac{d^2}{dx^2}}$$

which is (11.5) when generalized to 3D. The wave

Schrödinger Equation is Galilean invariant

Let use the transformation already discussed involving $\frac{\partial}{\partial x'} \to \frac{\partial}{\partial x'}$, $\frac{\partial}{\partial t'} \to \frac{\partial}{\partial t} + v\frac{\partial}{\partial x}$

assuming relative movement along x at speed v.

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x'^2}\psi'(x',t') + V(x')\psi'(x',t') = i\hbar\frac{\partial \psi'}{\partial t'}(x',t')$$

The potential V: if it depends on something "external" like caused by, say, a fixed "wall" then it is not invariant. Let us just assume it is a "two body" V like those used on showing the Newton's eqs. are Galilean inv. and forget about it, ok. $V(x_i'-x_j') = V(x_i - x_j)$. The focus will be on $\frac{\partial^2}{\partial x'^2}$ and on $\frac{\partial}{\partial t'}$.

Let use:

$$\psi'(x',t') = \psi(x,t)\underbrace{e^{-\frac{imvx}{\hbar}}}_{\gamma_1}\underbrace{e^{\frac{imv^2}{2\hbar}t}}_{\gamma_2}$$

$$\frac{\partial \psi'(x',t')}{\partial x'} \underset{\uparrow}{=} \frac{\partial \psi}{\partial x}\cdot\gamma_1\gamma_2 + \psi\left(\frac{-imv}{\hbar}\right)\gamma_1\gamma_2$$

$\frac{\partial}{\partial x'} = \frac{\partial}{\partial x}$

$$\frac{\partial^2 \psi'(x',t')}{\partial x'^2} = \frac{\partial^2 \psi}{\partial x^2}\cdot \gamma_1\gamma_2 + 2\left(\frac{\partial \psi}{\partial x}\right)\left(\frac{-imv}{\hbar}\right)\gamma_1\gamma_2 + \psi\left(\frac{-imv}{\hbar}\right)^2\gamma_1\gamma_2$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi'}{\partial x'^2}(x',t) = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}\cdot\gamma_1\gamma_2 + \left(-\frac{\hbar^2}{2m}\right)\left(\frac{-2imv}{\hbar}\right)\frac{\partial\psi}{\partial x}\cdot\gamma_1\gamma_2$$

$$+\left(-\frac{\hbar^2}{2m}\right)\left(\frac{-imv}{\hbar}\right)^2\gamma_1\gamma_2 =$$

$$= -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}\cdot\gamma_1\gamma_2 + i\hbar v\frac{\partial\psi}{\partial x}\cdot\gamma_1\gamma_2 + \frac{1}{2}mv^2\cdot\gamma_1\gamma_2$$

Then, $i\hbar\frac{\partial}{\partial t'}\psi'(x',t') = i\hbar\left(\frac{\partial}{\partial t} + v\frac{\partial}{\partial x}\right)\psi(x,t)\gamma_1\gamma_2 =$

$$= i\hbar\left[\frac{\partial\psi}{\partial t}\cdot\gamma_1\gamma_2 + \psi\gamma_1\left(\frac{imv^2}{2\hbar}\right)\gamma_2 + v\frac{\partial\psi}{\partial x}\cdot\gamma_1\gamma_2 + v\psi\left(\frac{-imv}{\hbar}\right)\gamma_1\gamma_2\right]$$

$$= i\hbar\frac{\partial\psi}{\partial t}\gamma_1\gamma_2 + \left(-\frac{1mv^2}{2}\right)\psi\gamma_1\gamma_2 + i\hbar v\frac{\partial\psi}{\partial x}\gamma_1\gamma_2 + mv^2\psi\gamma_1\gamma_2$$

$$+\frac{1}{2}mv^2\psi\gamma_1\gamma_2$$

$$= i\hbar\frac{\partial\psi}{\partial t}\cdot\gamma_1\gamma_2 + \underbrace{\frac{1}{2}mv^2\psi\gamma_1\gamma_2 + i\hbar v\frac{\partial\psi}{\partial x}\cdot\gamma_1\gamma_2}_{\text{Equal to those coming from } -\frac{\hbar^2}{2m}\frac{\partial^2\psi'}{\partial x'^2} \text{ thus they cancel.}}$$

Since $\gamma_1\gamma_2$ is common to both sides, they also drop.

Then, the invariance is shown to be true.

<u>Note</u>. Following the same procedure used to prove that (11.5) is <u>not</u> galilean invariant, can we show that it is <u>Lorentz</u> invariant?

Remember the identities we prove?

$$\frac{d}{dt'} = \frac{d}{dt} + v\frac{d}{dx} \quad \text{and} \quad \frac{d}{dx'} = \frac{d}{dx} \quad \text{if in 1D.}$$

this was for <u>galilean</u> transformations! Thus, we need to start with the "chain rule" again.

$$\frac{d}{dt'}\psi = \frac{dx}{dt'}\frac{d\psi}{dx} + \frac{dt}{dt'}\frac{d\psi}{dt} \underset{\substack{\uparrow \\ \text{dividing by } c}}{\Rightarrow} \frac{d\psi}{dx_0'} = \frac{dx}{dx_0'}\frac{d\psi}{dx} + \frac{dx_0}{dx_0'}\frac{d\psi}{dx_0}$$

Now use

$$x_0' = \gamma(x_0 - \beta x_1)$$
$$x_1' = \gamma(x_1 - \beta x_0)$$

First invert. This is done easily changing <u>primed</u> to <u>non primed</u> and viceversa and changing $\beta \to -\beta$.

$$x_0 = \gamma(x_0' + \beta x_1')$$
$$x_1 = \gamma(x_1' + \beta x_0')$$

Then $\frac{dx_1}{dx_0'} = \gamma\beta$, $\frac{dx_0}{dx_0'} = \gamma$ and we get

$$\frac{d}{dx_0'} = \gamma\beta\frac{d}{dx} + \gamma\frac{d}{dx_0}$$

Repeat for the other component:

$$\frac{d\psi}{dx'} = \frac{dx}{dx'}\frac{d\psi}{dx} + \frac{dt}{dx'}\frac{d\psi}{dt} \quad \text{or} \quad \frac{d\psi}{dx'} = \frac{dx}{dx'}\frac{d\psi}{dx} + \frac{dx_0}{dx'}\frac{d\psi}{dx_0}$$

i.e. for 1D:

$$\frac{d}{dx_1'} = \underbrace{\left(\frac{dx_1}{dx_1'}\right)}_{\gamma}\frac{d}{dx_1} + \underbrace{\left(\frac{dx_0}{dx_1'}\right)}_{\gamma\beta}\frac{d}{dx_0} = \gamma\frac{d}{dx_1} + \gamma\beta\frac{d}{dx_0}$$

Then

$$\boxed{\frac{d^2}{dx_1'^2} - \frac{d^2}{dx_0'^2}} = \left(\gamma\frac{d}{dx_1} + \gamma\beta\frac{d}{dx_0}\right)^2 - \left(\gamma\beta\frac{d}{dx_1} + \gamma\frac{d}{dx_0}\right)^2 =$$

$$= \gamma^2\frac{d^2}{dx_1^2} + \cancel{2\gamma^2\beta\frac{d}{dx_1}\frac{d}{dx_0}} + \gamma^2\beta^2\frac{d^2}{dx_0^2} - \gamma^2\beta^2\frac{d^2}{dx_1^2} - \cancel{2\gamma^2\beta\frac{d}{dx_1}\frac{d}{dx_0}} - \gamma^2\frac{d^2}{dx_0^2}$$

$$= \underbrace{\gamma^2(1-\beta^2)}_{1}\frac{d^2}{dx_1^2} - \underbrace{\gamma^2(1-\beta^2)}_{1}\frac{d^2}{dx_0^2} = \boxed{\frac{d^2}{dx_1^2} - \frac{d^2}{dx_0^2}}$$

Showing that indeed the wave equation is Lorentz invariant.

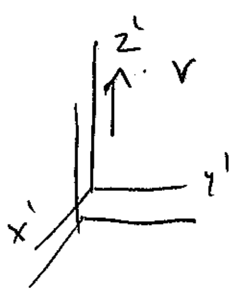
— ∘ —

11.3 Lorentz Transformations

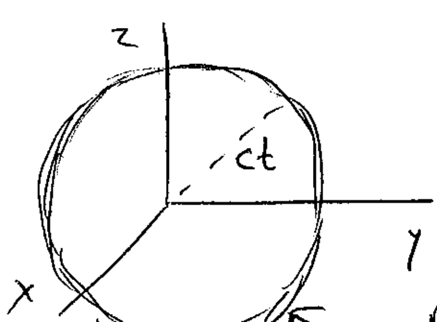
The requirement that the speed of light is constant <u>independent</u> of the motion of the source says that the galilean transformations can <u>not</u> be right (since the wave equation changes under a galilean transf, thus "c" can't be the same).

It was found that we need a different transf. which are the <u>Lorentz transformations</u>.

Consider two inertial reference frames K and K' with relative velocity $\vec{v}$. A point in K is (t, x, y, z) while a point in K' is (t', x', y', z'). For simplicity consider that K' moves with respect to K along the z axis. Consider also that $K = K'$ at $t = t' = 0$

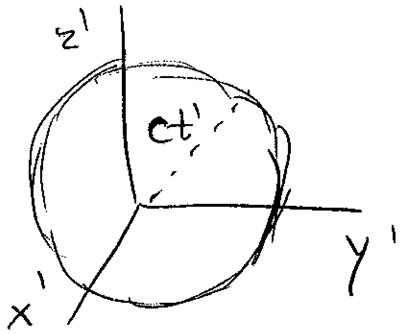


Suppose at $t = t' = 0$ a <u>pulse of light</u> is switched on and off. For K, the light reaches (x, y, z) at time t via the relation



$$(ct)^2 = x^2 + y^2 + z^2$$

Here comes the important point. From the perspective of K', the light moves at the <u>same</u> speed!! Thus, for K', even if moving with respect to K, the same relation holds but all "primed".



$$(ct')^2 = x'^2 + y'^2 + z'^2$$

↑ but c is <u>not</u> primed, but is the same as in K.

Thus, $\quad c^2t^2 - (x^2+y^2+z^2) = c^2t'^2 - (x'^2+y'^2+z'^2)$

and we have to find <u>linear</u> relations between $\underbrace{K \text{ and } K'}_{\text{the coordinates}}$ such that occurs. Instead of deducing them, let us try the following and see if they work:

$$\boxed{\begin{aligned} & ct' = x_0' = \gamma\,(\overset{ct}{x_0} - \beta z) \\ x_1' \to\ & z' = \gamma\,(z - \beta x_0) \\ x_2' \to\ & x' = x \\ x_3' \to\ & y' = y \end{aligned}}$$

with $\gamma = \dfrac{1}{\sqrt{1-\beta^2}}$ and $\beta = \dfrac{v}{c}$

$$(ct')^2 = x_0'^2 = \gamma^2 (x_0 - \beta z)^2 = \frac{1}{(1-\beta^2)} \left(x_0^2 - 2x_0 \beta z + \beta^2 z^2 \right)$$

$$z'^2 = \gamma^2 (z - \beta x_0)^2 = \frac{1}{1-\beta^2} \left(z^2 - 2z\beta x_0 + \beta^2 x_0^2 \right)$$

Then,

$$x_0'^2 - z'^2 = \frac{1}{1-\beta^2} \left[x_0^2 (1-\beta^2) + z^2 (\beta^2 - 1) \right] - \cancel{\frac{2x_0\beta z}{1-\beta^2}} + \cancel{\frac{2z x_0 \beta}{1-\beta^2}}$$

$$= x_0^2 - z^2$$

and of course $x'^2 + y'^2 = x^2 + y^2$

Then indeed it works! The relation between the coordinates of K and K' is called <u>Lorentz transformation</u>.

Note that z' and z are transformed and <u>also</u> t' and t, i.e. $t' \neq t$!

<u>Note</u>: Since β runs between 0 and 1 and γ goes from $\underset{(\beta=0)}{1}$ to $\underset{(\beta\to 1)}{\infty}$, then we can use a parametrization

$$\beta = \tanh(\xi) \qquad \beta\gamma = \sinh(\xi)$$
$$\gamma = \cosh(\xi)$$

and then

$$x_0' = \cosh(\xi)\, x_0 - \sinh(\xi)\, x_1$$

$$x_1' = \sinh(\xi)\, x_0 + \cosh(\xi)\, x_1$$

or

$$\boxed{\begin{pmatrix} x'_0 \\ x'_1 \\ x'_2 \\ x'_3 \end{pmatrix} = \left(\begin{array}{cc|cc} \cosh & -\sinh & 0 & 0 \\ -\sinh & \cosh & 0 & 0 \\ \hline 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right) \begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{pmatrix}}$$ which is (11.95).

The 2x2 block is similar to rotations of coordinates but now using hyperbolic functions.

4-vectors

In 3D we are familiar with the concept of vectors. they have 3 components (x_1, x_2, x_3) (or if not the coordinates then say (E_1, E_2, E_3) for vector $\vec{E}$). The 3D vectors transform the same as the coordinates i.e. if $\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \underbrace{\begin{pmatrix} 3\times 3 \end{pmatrix}}_{A} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$, then $\begin{pmatrix} E'_x \\ E'_y \\ E'_z \end{pmatrix} = A \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}$.

Let us generalize to 4D now! The "coordinate" 4-vector is (x_0, x_1, x_2, x_3). An arbitrary 4-vector will be (A_0, A_1, A_2, A_3). A set of 4 numbers is a 4-vector if under rotations they

Then:

$$\boxed{\begin{aligned} A_0' &= \gamma(A_0 - \beta A_1) \\ A_1' &= \gamma(A_1 - \beta A_0) \\ A_2' &= A_2 \\ A_3' &= A_3 \end{aligned}} \qquad (11.22)$$

for the case discussed before. Since for the coordinates we know that $X_0^2 - (X_1^2 + X_2^2 + X_3^2)$ remains the same under Lorentz transformations

then $A_0'^2 - (A_1'^2 + A_2'^2 + A_3'^2) = A_0^2 - (A_1^2 + A_2^2 + A_3^2)$ (11.23)

We can define the "scalar product" as

$A_0' B_0' - \vec{A}' \cdot \vec{B}'$. It can be shown that this is an invariant analogous to the lenght of a vector in 3D:

$$\boxed{A_0' B_0' - \vec{A}' \cdot \vec{B}' = A_0 B_0 - \vec{A} \cdot \vec{B}} \qquad (11.24)$$

Is (11.24) true ? **Proof:**

$$A_0'B_0' - \vec{A}' \cdot \vec{B}' = A_0'B_0' - (A_1'B_1' + A_2'B_2' + A_3'B_3')$$

$$= \gamma(A_0 - \beta A_1)\gamma(B_0 - \beta B_1) - \left[\gamma(A_1 - \beta A_0)\gamma(B_1 - \beta B_0) + A_2B_2 + A_3B_3\right] =$$

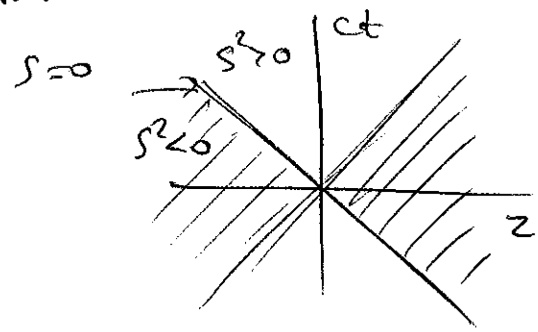
$$= \gamma^2\left[A_0B_0 - \cancel{\beta A_0B_1} - \cancel{\beta A_1B_0} + \beta^2 A_1B_1\right]$$
$$- \gamma^2\left[A_1B_1 - \cancel{\beta A_0B_1} - \cancel{\beta A_1B_0} + \beta^2 A_0B_0\right] - (A_2B_2 + A_3B_3)$$

$$= \underbrace{\gamma^2(1-\beta^2)}_{1} A_0B_0 + \underbrace{\gamma^2(\beta^2-1)}_{-1} A_1B_1 - (A_2B_2 + A_3B_3)$$

$$= A_0B_0 - (A_1B_1 + A_2B_2 + A_3B_3) = A_0B_0 - \vec{A} \cdot \vec{B}$$

Thus, (11.24) is indeed correct.

Note that the "norm" $ct^2 - (x^2+y^2+z^2) = s^2$ can be >0 or <0! In page 527 it is discussed that if s^2 we say that the origin $t=x=y=z=0$ and the arbitrary point (t,x,y,z) are "time like separated"



It is possible for an object to go from the origin to (t,x,y,z) as long as $s^2>0$. If $s^2<0$, then it is <u>not</u> possible.

In 11.5 it is shown that

$$\vec{p} = \frac{m\vec{u}}{\sqrt{1-\frac{u^2}{c^2}}} \quad ; \quad E = \frac{mc^2}{\sqrt{1-\frac{u^2}{c^2}}}$$

and $(p_0,\vec{p})$ is a 4-vector (with $p_0 = \frac{E}{c}$)

and $p_0^2 - \vec{p}\cdot\vec{p} = (mc)^2$ being the norm of the 4-vector then it is invariant. All this is not crucial for the main goal of showing that the Maxwell Eqs. are covariant.
