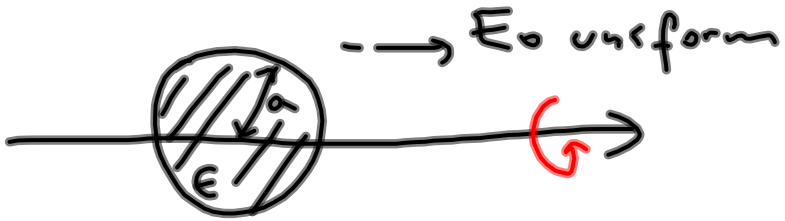


$$\phi_{inside} = \sum_{\ell} A_\ell r^\ell P_\ell(\cos\theta)$$

$$\phi_{outside} = -E_0 r\cos\theta + \sum_{\ell} C_\ell r^{-(\ell+1)} P_\ell(\cos\theta)$$

$$-\frac{1}{a}\frac{\partial \phi_{in}}{\partial \theta}\bigg|_{r=a} = -\frac{1}{a}\frac{\partial \phi_{out}}{\partial \theta}\bigg|_{r=a} ; \quad -\epsilon\frac{\partial \phi_{in}}{\partial r}\bigg|_{r=a} = -\epsilon_0\frac{\partial \phi_{out}}{\partial r}\bigg|_{r=a}$$

$$-\frac{1}{a}\sum_{\ell} A_\ell a^\ell \frac{\partial}{\partial\theta}P_\ell(\cos\theta) = -\frac{1}{a}\left[-E_0 a \frac{\partial}{\partial\theta}\overbrace{(\cos\theta)}^{P_1(\cos\theta)} + \sum_{\ell} C_\ell a^{-(\ell+1)}\frac{\partial}{\partial\theta}P_\ell(\cos\theta)\right]$$

$$\ell=1 \neq \ell\neq 1$$

$\ell = 1 \qquad A_1 = -E_0 + \frac{C_1}{a^3}$

$\ell \neq 1 \qquad A_\ell = \frac{C_\ell}{a^{2\ell+1}}$

$\frac{\epsilon}{\epsilon_0} A_1 = -E_0 - 2\frac{C_1}{a^3}$

$\frac{\epsilon}{\epsilon_0} \ell A_\ell = -\frac{C_\ell}{a^{2\ell+1}}(\ell+1)$

Solution: $A_\ell = C_\ell = 0, \ \ell \neq 1$

$$A_1 = -\left(\frac{3}{2+\frac{\epsilon}{\epsilon_0}}\right)E_0 \ ; \quad C_1 = \left(\frac{\frac{\epsilon}{\epsilon_0}-1}{\frac{\epsilon}{\epsilon_0}+2}\right)a^3 E_0$$

$$\phi_{in} = -\left(\frac{3}{2+\frac{\epsilon}{\epsilon_0}}\right)E_0 r\cos\theta \ ; \quad \phi_{out} = -E_0 r\cos\theta + \left(\frac{\frac{\epsilon}{\epsilon_0}-1}{\frac{\epsilon}{\epsilon_0}+2}\right)E_0 \frac{a^3}{r^2}\cos\theta$$

$$E_{inside} = \frac{3}{2+\frac{\epsilon}{\epsilon_0}} E_0 < E_0$$

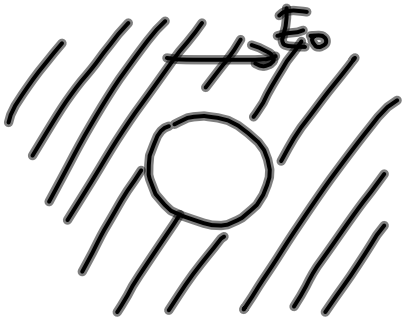
$$\xrightarrow[\epsilon \to \infty]{} 0$$

$$\vec{E} = E_0 + E_{dipole}$$

$$\vec{P}_{in} = \underbrace{(\epsilon - \epsilon_0)}_{\epsilon_0 \chi_e} \vec{E} = 3\epsilon_0 \left(\frac{\frac{\epsilon}{\epsilon_0} - 1}{\frac{\epsilon}{\epsilon_0} + 2} \right) \vec{E}_0$$

$$\vec{P}_{out} = 0$$

From $\Delta \vec{P}$ at surface $\longrightarrow \sigma_{pol} = 3\epsilon_0 \left(\frac{\frac{\epsilon}{\epsilon_0} - 1}{\frac{\epsilon}{\epsilon_0} + 2} \right) E_0 \cos\theta$.



$\epsilon_0 \to \epsilon$

$\epsilon \to \epsilon_0$

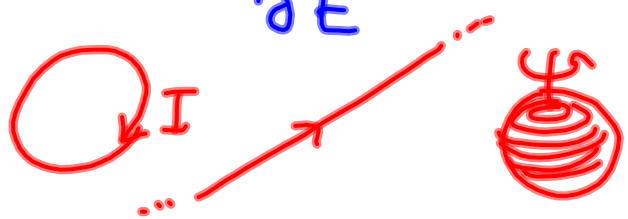
$$E_{in} = \frac{3E_0}{2 + \frac{\epsilon_0}{\epsilon}} > E_0$$

ch. 5 Magnetostatics

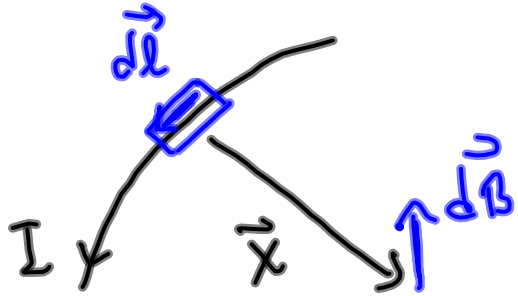
5.1 currents and magnetic fields

Continuity Eq. $\nabla \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0$

$$\boxed{\nabla \cdot \vec{J} = 0}$$

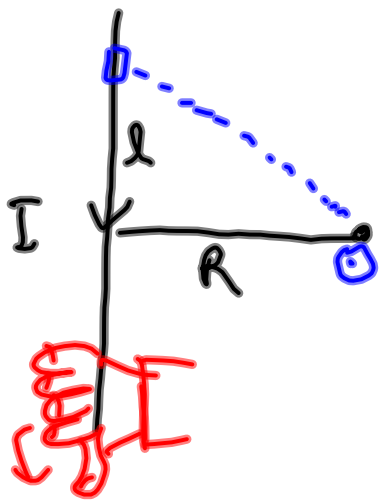


5.2 Biot-Savart law



$$d\vec{B} = \frac{\mu_0}{4\pi} I \left(\frac{d\vec{l} \times \vec{x}}{|\vec{x}|^3} \right)$$

μ_0 = permeability of space



$$|\vec{B}| = \frac{\mu_0}{4\pi} I \int_{-\infty}^{+\infty} \frac{dl\, R}{(R^2 + l^2)^{3/2}} = \frac{\mu_0 I}{2\pi R}$$

$$\vec{F} = q(\cancel{\vec{E}} + \vec{v}\times\vec{B}) \quad , \quad d\vec{F} = \underbrace{\overset{I}{\frac{dq}{dt}}}(d\vec{l}\times\vec{B})$$

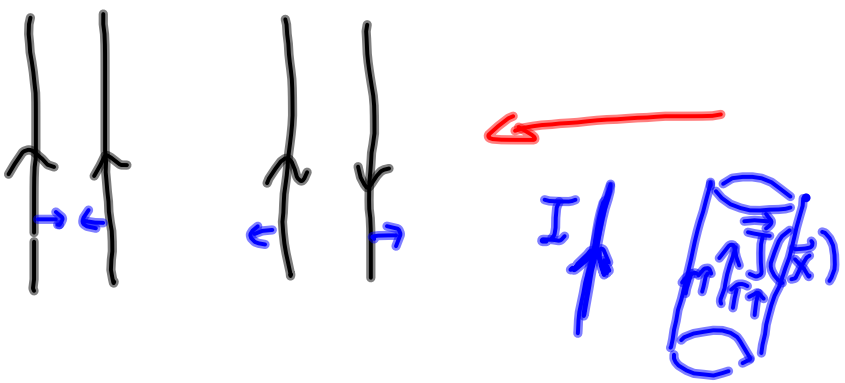
$$\vec{v} = \frac{d\vec{l}}{dt}$$

B-S law

$$\vec{F}_{12} = \oint_1 I_1 \left(d\vec{l}_1 \times \vec{B}_{\text{caused by 2}}\right) =$$

on 1 caused by 2

$$= \frac{\mu_0 I_1 I_2}{4\pi} \oint_1 \oint_2 d\vec{l}_1 \times \left(\frac{d\vec{l}_2 \times \vec{x}_{12}}{|\vec{x}_{12}|^3}\right)$$



$$\nabla_x \times \vec{B}(\vec{x}) = \frac{\mu_0}{4\pi}\int\left[\vec{J}(\vec{x}') \times \underbrace{\frac{(\vec{x}-\vec{x}')}{|\vec{x}-\vec{x}'|^3}}_{-\nabla_x\left(\frac{1}{|\vec{x}-\vec{x}'|}\right)}\right] d^3x' =$$

$$\boxed{\nabla\cdot\vec{B} = 0}$$

$$= \frac{\mu_0}{4\pi}\nabla_x\left[\nabla_x \times \int \frac{\vec{J}(\vec{x}')}{|\vec{x}-\vec{x}'|} d^3x'\right] = 0$$

$$\nabla\times(\nabla\times\vec{a}) = \nabla(\nabla\cdot\vec{a}) - \nabla^2\vec{a}$$

$$\nabla^2\left(\frac{1}{|\vec{x}-\vec{x}'|}\right) = -4\pi\delta(\vec{x}-\vec{x}')$$

$$\boxed{\nabla\times\vec{B} = \mu_0\vec{J}}$$

$$\nabla\times\vec{E} = 0, \quad \nabla\cdot\vec{E} = \frac{\rho}{\epsilon_0}$$

Stoke's theorem

$$\int_S (\nabla \times \vec{B}) \cdot \vec{n}\, da \overset{\text{physics}}{=} \int_S \mu_0 \vec{J} \cdot \vec{n}\, da$$

$$\overset{\text{math}}{=} \oint_C \vec{B} \cdot d\vec{l}$$

$$\boxed{\oint_C \vec{B} \cdot d\vec{l} = \mu_0 \int_S \vec{J} \cdot \vec{n}\, da = \mu_0 I}$$

Ampere's law