Biot-Savart

Forces

Ampere's law

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$$\nabla \cdot \vec{B} = 0 \,, \quad \nabla \cdot \vec{J} = 0$$

$$\nabla \times \vec{B} = \mu_0 \vec{J}$$

$$\nabla \times \vec{E} = 0$$

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

5.4 Vector Potential

$$\nabla \cdot \vec{B} = 0 \,, \quad \vec{B} = \nabla \times \vec{A}$$

vector potential

$$\vec{B}(\vec{x}) = \nabla \times \underbrace{\frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{x}')}{|\vec{x}-\vec{x}'|} d^3x'}_{\vec{A}(\vec{x})}$$

$$\phi(\vec{x}) = \int \frac{\rho(\vec{x}')}{|\vec{x}-\vec{x}'|} d^3x'$$

$$\vec{A} = \frac{\mu_0}{4\pi} \int \frac{J(x')}{|\vec{x}-\vec{x}'|} d^3x' + \nabla \psi(\vec{x})$$

$$\{A\} \to \{A'\} \to \{A''\} \to \cdots$$

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$$\nabla\cdot\vec{A} = \frac{\mu_0}{4\pi}\int \vec{J}(\vec{x}')\cdot\underbrace{\nabla\left(\frac{1}{|\vec{x}-\vec{x}'|}\right)}_{-\nabla'\left(\frac{1}{|\vec{x}-\vec{x}'|}\right)} d^3x' + \nabla^2\psi = \nabla^2\psi$$

$\vec{x}$

$$\underbrace{\nabla'\cdot\left[\frac{\vec{J}(\vec{x}')}{|\vec{x}-\vec{x}'|}\right]}_{\int_V \nabla'\cdot[\ ] = \int_S [\ ]\cdot\vec{n}\,dS \to 0} = \frac{\underbrace{[\nabla'\cdot\vec{J}(\vec{x}')]}_{=0}}{|\vec{x}-\vec{x}'|} + \vec{J}(\vec{x}')\cdot\nabla'\left(\frac{1}{|\vec{x}-\vec{x}'|}\right)$$

local $\vec{J}$

$$\nabla\cdot\vec{A} = 0 \quad \text{Coulomb gauge}$$

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$$\nabla \times \vec{B} = \mu_0 \vec{J} = \nabla \times (\nabla \times \vec{A}) = \nabla \underbrace{(\nabla \cdot \vec{A})}_{=0} - \nabla^2 \vec{A}$$

$$\boxed{\nabla^2 \vec{A} = -\mu_0 \vec{J}}$$

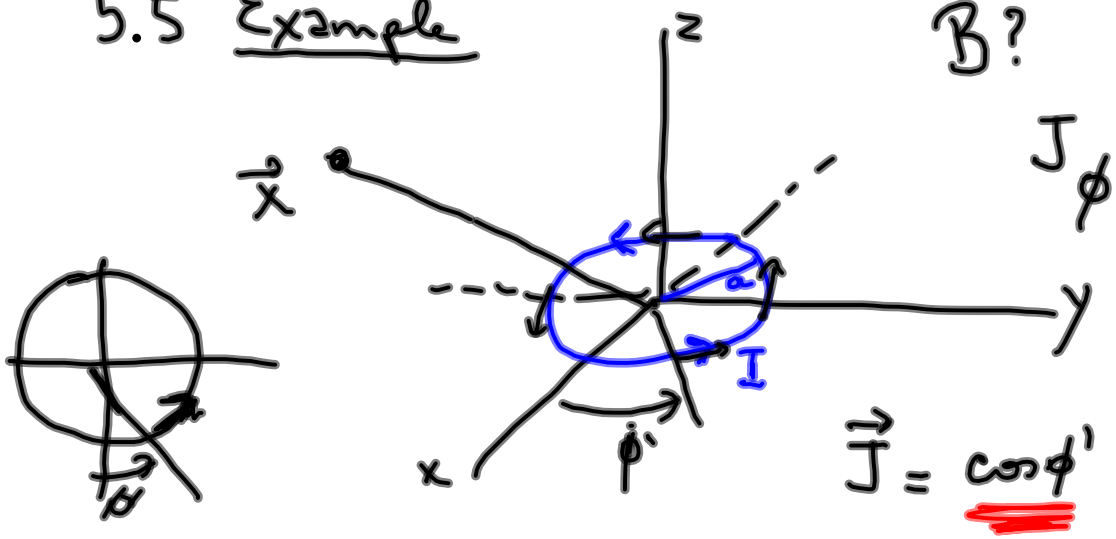
(if $\nabla \cdot \vec{A} = 0$)

$$\nabla^2 \phi = -\frac{\rho}{\epsilon_0}$$

5.5 Example

B?

$$\int \underbrace{dx}_{L} \underbrace{\delta(x)}_{1/L} = 1$$

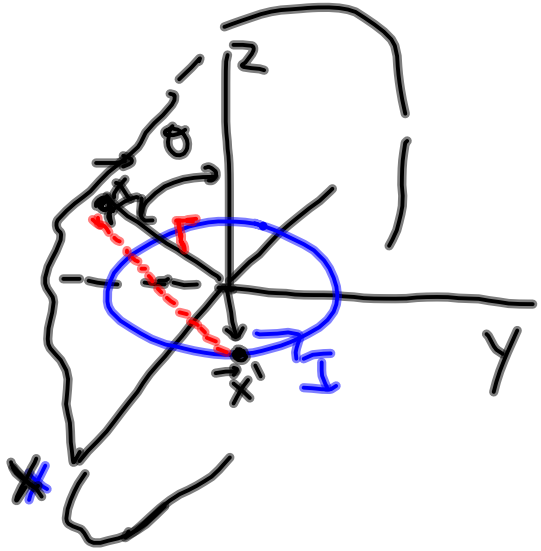


$$J_\phi = \frac{I}{a} \delta(\cos\theta') \delta(r' - a)$$

$\theta' = \pi/2$ $\quad r' = a$

$$[J] = \frac{[I]}{L^2}$$

$$\vec{J} = \cos\phi' J_\phi \hat{e}_y - \sin\phi' J_\phi \hat{e}_x$$



$$\vec{x} = (r\sin\theta,\ 0,\ r\cos\theta)$$

$$\vec{x}' = (a\cos\phi',\ a\sin\phi',\ 0)$$

$$|\vec{x}-\vec{x}'| = \sqrt{(r\sin\theta - a\cos\phi')^2 + a^2\sin^2\phi' + r^2\cos^2\theta}$$

$$= \sqrt{r^2 + a^2 - 2ra\sin\theta\cos\phi'}$$

$$\underbrace{\int_0^{2\pi} d\phi' \frac{\overset{J}{\sin\phi'}}{\sqrt{}}}_{=0} \quad ; \quad \int d\phi' \frac{\cos\phi'}{\sqrt{}}$$

$$\vec{A} = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(\vec{x}')}{|\vec{x}-\vec{x}'|} d^3x \Rightarrow$$

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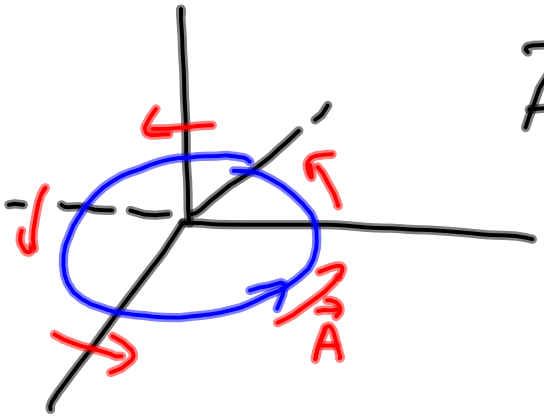
$$\boxed{A_\phi = \frac{\mu_0 I a}{4\pi}\int_0^{2\pi} \frac{\cos\phi'\, d\phi'}{\sqrt{r^2+a^2-2ra\sin\theta\cos\phi'}}}$$

$A_r = 0$

$A_\theta = 0$

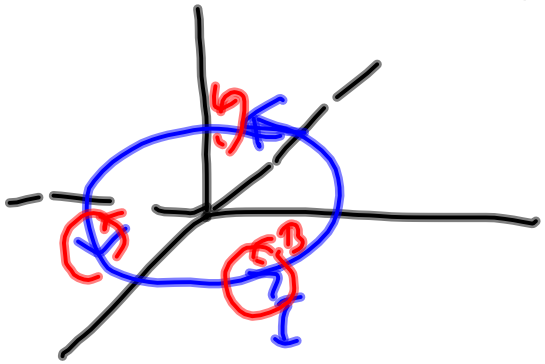


$$\vec{A} \sim \int \frac{\vec{J}}{|\vec{x}-\vec{x}'|}$$

$$\vec{B} = \nabla \times \vec{A} =$$

$$= \hat{e}_r \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta A_\phi)$$

$$+ \hat{e}_\theta \left(-\frac{1}{r}\frac{\partial}{\partial r}(r A_\phi)\right)$$



$\underline{r \gg a}$

$$\sqrt{r^2+a^2-2ra\sin\theta\cos\phi'} \approx r$$

$$\frac{1}{\sqrt{r^2+a^2-\underline{2ra\sin\theta\cos\phi'}}} \approx \frac{1}{r}\left(1+\frac{a}{r}\sin\theta\cos\phi'\right)$$

A.I

$$A_\phi \cong \frac{\mu_0 I a}{4\pi}\int_0^{2\pi}\cos\phi'\,d\phi'\;\frac{1}{r}\left(\cancel{1}+\frac{a}{r}\sin\theta\cos\phi'\right)$$

$$= \frac{\mu_0}{4\pi}\frac{I\pi a^2}{r^2}\sin\theta$$

$$\boxed{B_r = \frac{\mu_0}{2\pi}\frac{(Ia^2\pi)}{r^3}\cos\theta \qquad B_\theta = \frac{\mu_0}{4\pi}\frac{(Ia^2\pi)}{r^3}\sin\theta}$$

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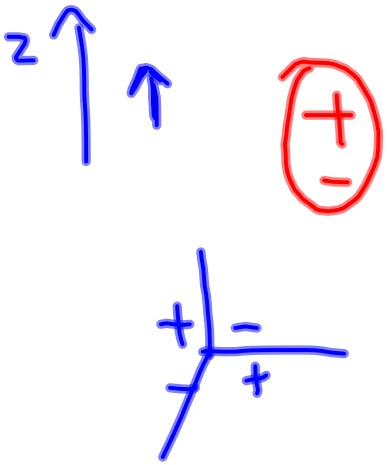
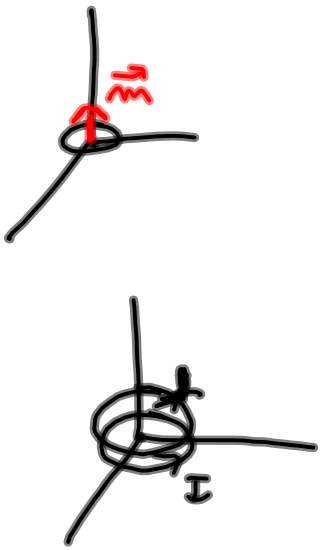
$$E_{dipole} \sim \frac{1}{2\pi\epsilon_0} \cdot \frac{p\cos\theta}{r^3}$$



$Q \neq 0$

$p = 0$

$q = 0$



magnetic dipole moment

$m = 0$