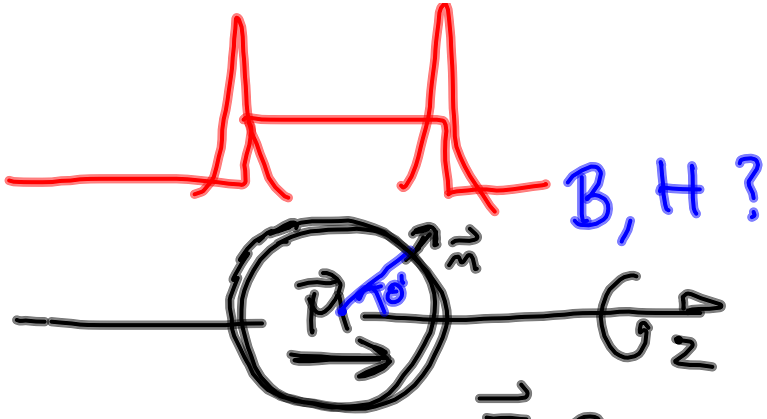
5.10



B, H ?

$\nabla\cdot\vec{M}$

$\vec{J}=0$

• P

$$\vec{M}=M_0\,\hat{e}_3$$

$$\sigma_M=\vec{n}\cdot\vec{M}$$

$$\sqrt{\frac{4\pi}{3}}\,\underline{Y_{10}(\theta',\phi')}$$

$$M_0\cos\theta'$$

ⓒ

$$\Phi_M=\cancel{\int_V\frac{\nabla\cdot J}{|\vec{x}-\vec{x}'|}}+\frac{1}{4\pi}\oint_{\text{sphere}}\frac{\vec{n}'\cdot\vec{M}}{|\vec{x}-\vec{x}'|}\,da'$$

$$\frac{1}{|\vec{x}-\vec{x}'|}=4\pi\sum_{l,m}\frac{1}{2l+1}\frac{r_<^{l}}{r_>^{l+1}}\,\underline{Y_{lm}^{*}(\theta',\phi')}\,Y_{lm}(\theta,\phi)$$

$$\Phi_M = \sum_{l,m} M_0 a^2 \frac{r_<^l}{r_>^{l+1}} \frac{1}{(2l+1)} Y_{lm}(\theta,\phi)\sqrt{\frac{4\pi}{3}}\,\delta_{l,1}\delta_{m,0}$$

$$= M_0 a^2 \frac{r_<}{r_>^2}\frac{1}{3}\underbrace{Y_{10}(\theta,\phi)\sqrt{\frac{4\pi}{3}}}_{\cos\theta} = \frac{M_0 a^2}{3}\frac{r_<}{r_>^2}\cos\theta$$

$$\Phi_M^{\text{inside}}(r,\theta,\phi) \overset{r_<=r,\; r_>=a}{=} \boxed{\frac{M_0 r\cos\theta}{3}} = \boxed{\frac{M_0 z}{3}}$$

$$\Phi_M^{\text{outside}}(r,\theta,\phi) = \boxed{\frac{M_0}{3}\frac{a^3\cos\theta}{r^2}\cdot\frac{r}{r}} = \boxed{\frac{M_0}{3}\left(\frac{a}{r}\right)^3 z}$$

Inside

$\Phi_M \, , \vec{H} = -\nabla \Phi_M$

$\boxed{H_z = -\frac{M_0}{3}} \, , \vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$

$$\vec{B} = \mu_0 (\vec{H} + \vec{m})$$

$$= \mu_0 \left(-\frac{\vec{M}}{3} + \vec{M} \right)$$

$$= \boxed{\mu_0 \frac{2}{3} \vec{M}}$$

Outside

$\Phi_M = \frac{1}{3} M_0 a^3 \frac{z}{r^3}$) example

$\Phi_M = \frac{1}{4\pi} \frac{\vec{m} \cdot \vec{x}}{r^3} + \cdots$) general

$$m_z = \frac{4\pi}{3} M_0 a^3 = M_0 . \text{volume}$$

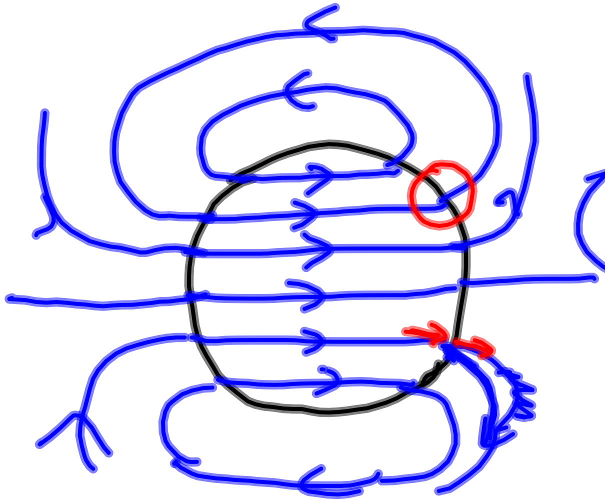
Outside

$$\vec{B} = \mu_0(\vec{H} + \vec{M})$$

$\vec{M}_{outside} = 0$

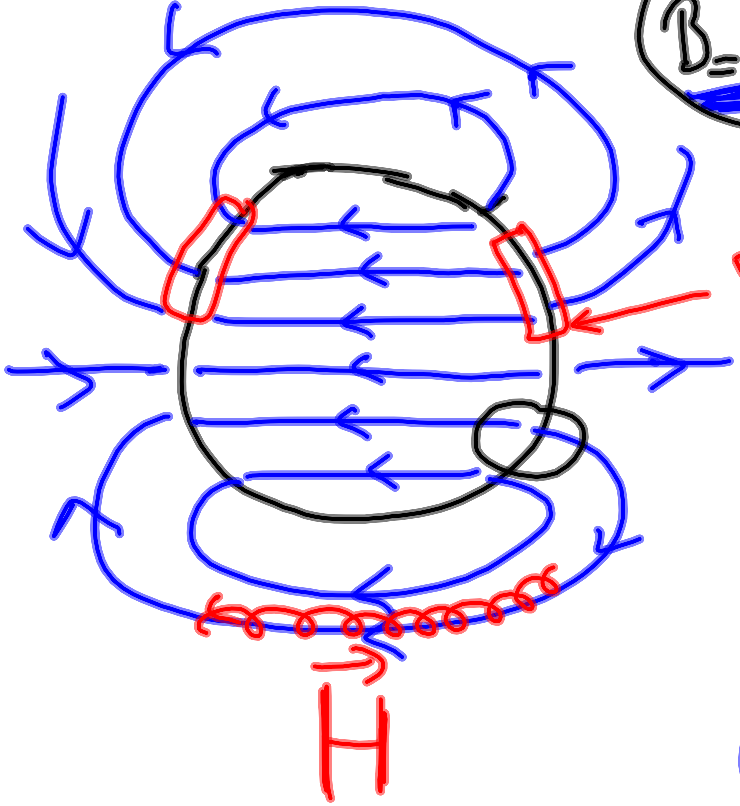
$-\nabla\Phi_M$, $\Phi_M = \frac{1}{3} M_0 a^3 \frac{\cos\theta}{r^2}$

$\nabla\left(\frac{\cos\theta}{r^2}\right)$ back of the book



$$\boxed{\nabla\cdot\vec{B}=0}$$

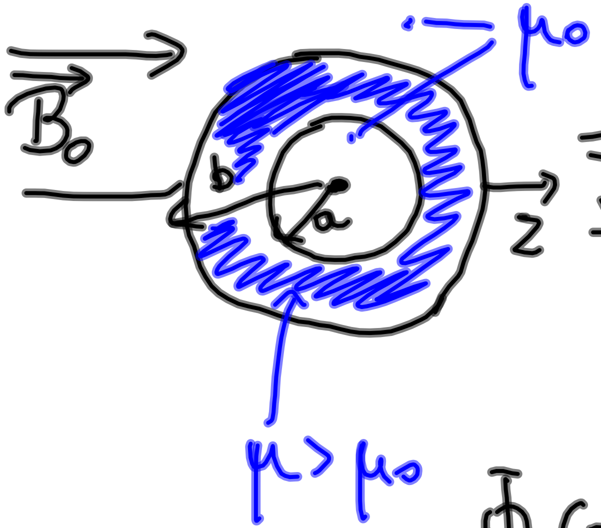
$\boxed{B=\mu_0 H}$ outside



$\nabla\cdot\vec{M}$

NASA

5.12 Magnetic shielding



$\vec{B}, \vec{H}$ everywhere ?

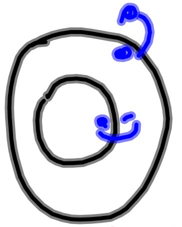
$\vec{J} = 0$

$\vec{H} = -\nabla \boxed{\Phi_M}$

$$\Phi_M(r,\theta) = \sum_{l=0}^{\infty}\left[A_l r^l + B_l \frac{1}{r^{l+1}}\right] P_l(\cos\theta)$$

$$\underset{r>b}{\Phi_M} = \underbrace{-H_0 r\cos\theta}_{-\nabla(-H_0 \underbrace{r\cos\theta}_{z})} + \sum_{l=0}^{\infty} \frac{\boxed{\alpha_l}}{r^{l+1}} P_l(\cos\theta)$$

$$\underset{a<r<b}{\Phi} = \sum_{l=0}^{\infty} \left(\boxed{\beta_l}\, r^l + \frac{\boxed{\gamma_l}}{r^{l+1}} \right) P_l(\cos\theta)$$



$$\underset{r<a}{\Phi_M} = \sum_{l=0}^{\infty} \boxed{\delta_l}\, r^l P_l(\cos\theta)$$

$$\left[\begin{array}{l} (\vec{B}_1 - \vec{B}_2)\cdot \vec{n}_{21} = 0 \\ \vec{n}_{21} \times (\vec{H}_2 - \vec{H}_1) = 0 \end{array} \right.$$

$\uparrow -\nabla \Phi_M$ → $\hat{e}_r$, → $\boxed{e_\theta}$, → e_ϕ

$$B = \tilde{\mu} H$$

$\mu \quad a<r<b$

$\mu_0 \quad r>b$
$\qquad r<a$

$$\left.\frac{\partial \Phi_m}{\partial \theta}\right|_b^{r>b} = \left.\frac{\partial \Phi_m}{\partial \theta}\right|_b^{a<r<b} \; ; \; \left.\frac{\partial \Phi_n}{\partial \sigma}\right|_a^{a<r<b} = \left.\frac{\partial \Phi_n}{\partial \sigma}\right|_a^{r<a} \qquad \text{“}\vec{H}\text{”}$$

$$\mu_0 \left.\frac{\partial \Phi_m}{\partial r}\right|_b^{r>b} = \mu \left.\frac{\partial \Phi_m}{\partial r}\right|_b^{a<r<b} \; ; \; \mu \left.\frac{\partial \Phi_n}{\partial r}\right|_a^{a<r<b} = \mu_0 \left.\frac{\partial \Phi_n}{\partial r}\right|_a^{r<a} \qquad \text{“}\vec{B}\text{”}$$

$\ell = 1$ nonzero

$\ell \neq 1$ zero

$\delta_\ell, \ \delta_1 \sim \frac{1}{(\mu/\mu_0)}$ at large $\frac{\mu}{\mu_0}$