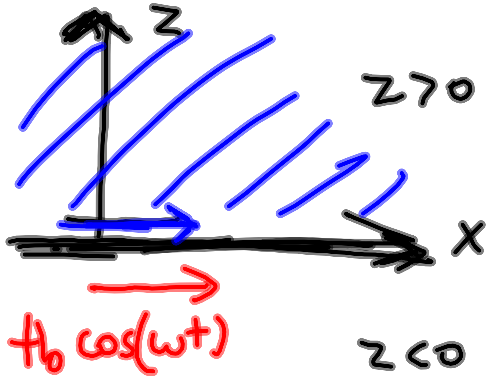


$$\left.\begin{aligned} &\nabla\times\vec{H} = \vec{J} \\ &\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \\ &\vec{H} = \frac{\vec{B}}{\mu} \\ &\vec{J} = \sigma\vec{E} \\ &\nabla\cdot\vec{A} = 0 \end{aligned}\right\} \quad \nabla^2\vec{A} = \mu\sigma\frac{\partial\vec{A}}{\partial t}$$

$+$ B. Cond.

$$\vec{A} = \mu\, y\, H(z)\,\hat{e}_z$$

$$\underbrace{\nabla\times\vec{A}}_{\vec{B}} = \mu\, H(z)\,\hat{e}_x$$

$$\boxed{\frac{\partial^2 H(z,t)}{\partial z^2} = \mu\sigma\frac{\partial H(z,t)}{\partial t}}$$

$$H(z,t) = h(z)\, e^{-i\omega t}$$

$$\left(\frac{\partial^2}{\partial z^2} + i\omega\mu\sigma\right) h(z) = 0 \quad , \quad \frac{\partial H(z,t)}{\partial t} = -i\omega H(z,t)$$

$$h(z) = e^{ikz}$$

$$\frac{\partial^2 h(z)}{\partial z^2} = -k^2 e^{ikz}$$

$$k^2 = i\omega\mu\sigma$$

$$k = \pm\sqrt{i}\sqrt{\omega\mu\sigma}$$

$$\sqrt{i} = e^{i\frac{\pi}{4}} = \frac{1}{\sqrt{2}}(1+i)$$

$$h(z) = e^{\pm i(1+i)\sqrt{\frac{\omega\mu\sigma}{2}}z} = e^{\pm(i-1)\sqrt{\cdots}\,z}$$

$$\sqrt{\cdots} \to 1/\delta$$

(+) physical solution

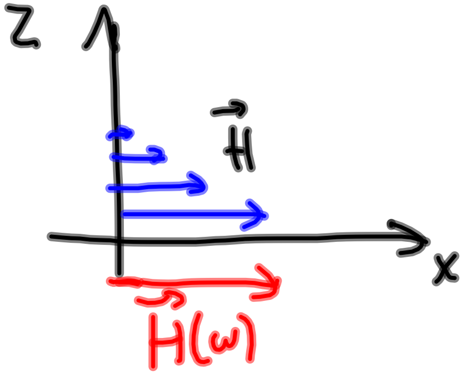
$$\delta = \sqrt{\frac{2}{\mu\sigma\omega}}$$

$$h(z) = e^{-z/\delta}\, e^{iz/\delta}$$

$$\boxed{H(z,t) = A\, e^{-z/\delta}\, e^{i\left(\frac{z}{\delta} - \omega t\right)}}$$

H along the x axis.

$$\underline{\delta = \text{skin depth}} = \sqrt{\frac{2}{\mu\sigma\omega}} \quad \begin{array}{l} \rightarrow \text{if } \sigma = \infty,\ \delta = 0 \\ \rightarrow \text{if } \omega = 0,\ \delta \to \infty \end{array}$$

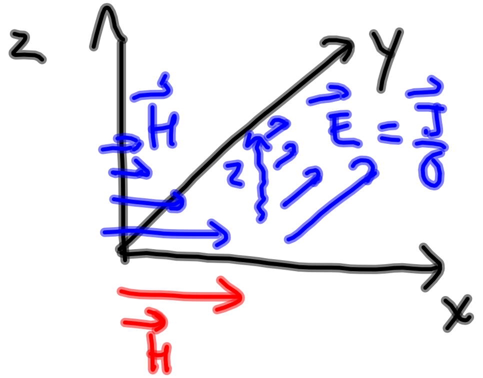


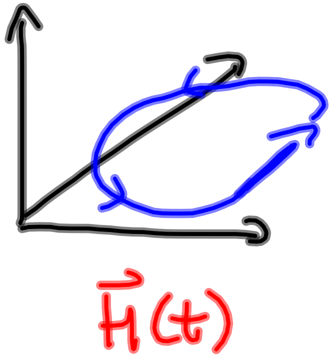
$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\underbrace{\nabla \times \vec{H}}_{} = \vec{J} = \sigma \vec{E} \quad \hookrightarrow (0, E_y, 0)$$

$$\begin{vmatrix} \hat{e}_x & \hat{e}_y & \hat{e}_z \\ \nabla_x & \nabla_y & \nabla_z \\ H_x & 0 & 0 \end{vmatrix} = \hat{e}_y \underbrace{\nabla_z H_x}_{\neq 0} - \hat{e}_z \underbrace{\nabla_y H_x}_{= 0}$$

$$\boxed{E_y = \frac{\cancel{\mu}\,\delta\,\cancel{\omega}}{\sqrt{2}} H_0\, e^{-z/\delta} \cos\left(\frac{z}{\delta} - \omega t + \frac{3\pi}{4}\right)}$$





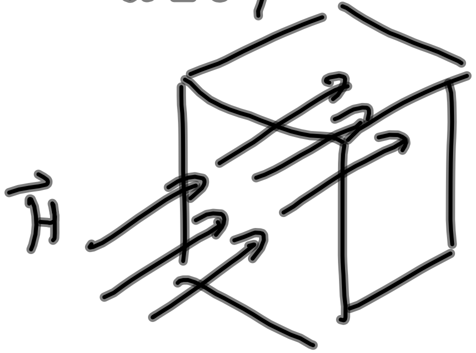
$\nabla \cdot \vec{J} = 0$

σ finite

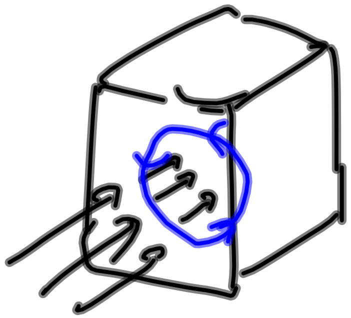
$\omega = 0, \delta \to \infty$

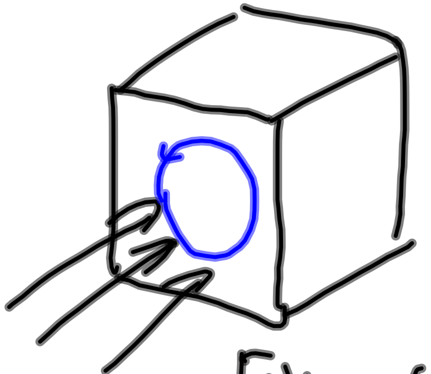
$\omega =$ finite

$\omega \to \infty, \delta \to 0$



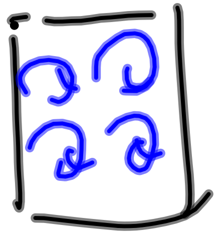
static





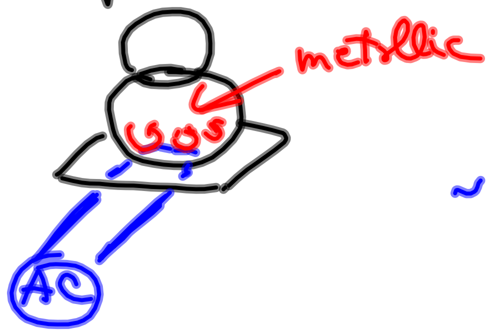
$E_y \sim \omega e^{-x/\delta} \ldots$

eddy currents



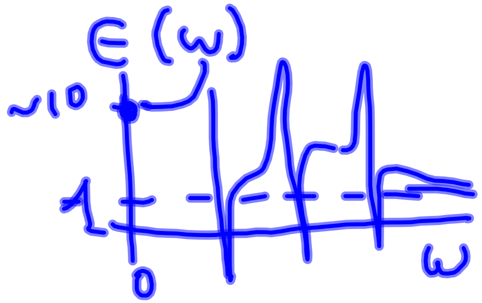
eddy current brakes

induction heating
(flameless cooking)



$\vec{J} = \sigma \vec{E}$

$\cancel{\vec{J} = \sigma(\omega) \vec{E}}$



6.1 Maxwell Equations

$$\nabla\cdot\vec{D}=\rho$$

$$\nabla\times\vec{H}=\vec{J}$$

$$\nabla\times\vec{E}+\frac{\partial\vec{B}}{\partial t}=0$$

$$\nabla\cdot\vec{B}=0$$

$$(\nabla\cdot\vec{J}=0)$$

$$\nabla\cdot\vec{J}+\frac{\partial\rho}{\partial t}=0$$

$$\nabla\cdot\vec{J}+\frac{\partial}{\partial t}(\nabla\cdot\vec{D})=0$$

$$\nabla\cdot\left(\vec{J}+\frac{\partial\vec{D}}{\partial t}\right)=0$$

$$\nabla\times\vec{H}=\vec{J}+\frac{\partial\vec{D}}{\partial t}$$

$$\nabla\cdot\vec{B}=0 \longrightarrow \boxed{\vec{B}=\nabla\times\vec{A}}$$

$$\nabla\times\vec{E}+\frac{\partial\vec{B}}{\partial t}=0\,,\quad \nabla\times\vec{E}+\frac{\partial}{\partial t}(\nabla\times\vec{A})=0$$

$$\nabla\times\underbrace{\left(\vec{E}+\frac{\partial\vec{A}}{\partial t}\right)}_{-\nabla\Phi}=0$$

$$\boxed{\vec{E}=-\nabla\Phi-\frac{\partial\vec{A}}{\partial t}}$$

$\nabla\cdot\vec{D}=\rho$ and $\nabla\times\vec{H}=\vec{J}+\dfrac{\partial\vec{D}}{\partial t}$