6.2 Vector and scalar potentials

$$\nabla\cdot\vec{B}=0 \longrightarrow \boxed{\vec{B}=\nabla\times\vec{A}}$$

$$\nabla\times\vec{E}+\frac{\partial\vec{B}}{\partial t}=0\ ,\quad \nabla\times\Big(\underbrace{\vec{E}+\frac{\partial\vec{A}}{\partial t}}_{-\nabla\Phi}\Big)=0$$

$$\boxed{\vec{E}=-\nabla\Phi-\frac{\partial\vec{A}}{\partial t}}$$

$$\nabla \cdot \vec{D} = \rho$$

$$\epsilon_0 \nabla \cdot \vec{E} = \rho$$

$$\boxed{\begin{array}{l} \vec{D} = \epsilon_0 \vec{E} \\ \vec{B} = \mu_0 \vec{H} \end{array}}$$

$$\epsilon_0 \nabla \cdot \left(-\nabla \Phi - \frac{\partial \vec{A}}{\partial t} \right) = \rho$$

$$\boxed{\nabla^2 \Phi + \frac{\partial}{\partial t} (\nabla \cdot \vec{A}) = -\frac{\rho}{\epsilon_0}}$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$$\frac{1}{\mu_0} \underbrace{\nabla \times (\nabla \times \vec{A})}_{\nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A}} = \vec{J} + \epsilon_0 \frac{\partial}{\partial t} \left(-\nabla \Phi - \frac{\partial \vec{A}}{\partial t} \right)$$

$$\boxed{\nabla^2\vec{A} - \frac{1}{c^2}\frac{\partial^2\vec{A}}{\partial t^2} - \nabla\underbrace{\left(\nabla\cdot\vec{A} + \frac{1}{c^2}\frac{\partial\Phi}{\partial t}\right)}_{=0} = -\mu_0\vec{J}}$$

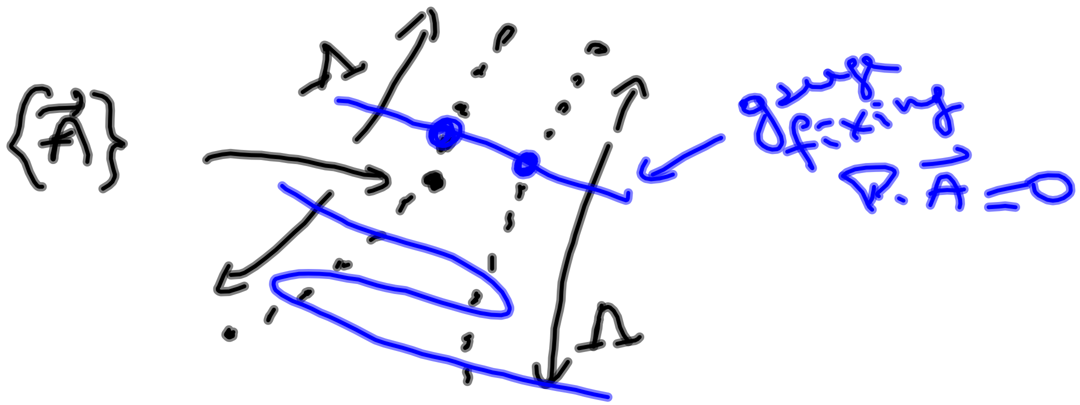
$$\boxed{\mu_0\epsilon_0 = \frac{1}{c^2}}$$

$$\nabla\times\vec{A}' = \nabla\times\vec{A} + \overbrace{\nabla\times\nabla\Lambda(\vec{x},t)}^{=0}$$

$$\Phi' = \Phi - \frac{\partial\Lambda(\vec{x},t)}{\partial t}$$

$$\nabla\cdot\vec{A} = 0$$

$$(\nabla\cdot\vec{A})^2 + \dots = 0$$



$$\nabla^2\Phi + \frac{\partial}{\partial t}\underbrace{(\nabla\cdot\vec{A})}_{-\frac{1}{c^2}\frac{\partial\Phi}{\partial t}} = -\frac{\rho}{\epsilon_0}$$

$-\frac{1}{c^2}\frac{\partial\Phi}{\partial t}$ if Lorentz gauge is used

$$\boxed{\begin{aligned} \frac{\partial}{\partial t}\nabla^2\Phi - \frac{1}{c^2}\frac{\partial}{\partial t}\frac{\partial^2\Phi}{\partial t^2} &= -\overbrace{\frac{\partial}{\partial t}\rho}^{\nabla\cdot\vec{J}}\Big/\epsilon_0 \\ \nabla^2\vec{A} - \frac{1}{c^2}\frac{\partial^2\vec{A}}{\partial t^2} &= -\mu_0\vec{J} \end{aligned}}$$

$$\underline{\nabla\cdot\vec{J} + \frac{\partial\rho}{\partial t} = 0}$$

$$\underline{\nabla \cdot \vec{A} = 0} \rightsquigarrow \nabla^2 \Phi(\vec{x},t) = -\frac{\rho}{\epsilon_0}(\vec{x},t)$$

$$\Phi(\vec{x},t) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{x}',t)\, d^3x'}{|\vec{x}-\vec{x}'|}$$

"instantaneous Coulomb potential"

$$\vec{E} = -\nabla\Phi - \frac{\partial \vec{A}}{\partial t}$$

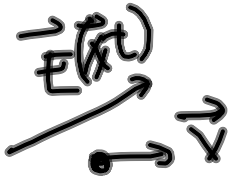
$$\boxed{\nabla^2 \vec{A} - \frac{1}{c^2}\frac{\partial^2 \vec{A}}{\partial t^2}} = -\mu_0 \vec{J} + \frac{1}{c^2}\nabla\left(\frac{\partial \Phi}{\partial t}\right) = \boxed{-\mu_0 \vec{J}_t}$$

$$\vec{J} = \vec{J}_{\text{longitudinal}} + \vec{J}_{\text{transverse}}$$

$$\nabla \times \vec{J}_l = 0 \qquad \nabla \cdot \vec{J}_t = 0$$

$$\vec{J}_t = \frac{1}{4\pi}\nabla \times \left[\nabla \times \int \frac{\vec{J}(\vec{x}',t)\, d^3x'}{|\vec{x}-\vec{x}'|}\right]$$

6.7 Conservation of Energy



$$W_{A\to B} = -q\int_{A\to B}\vec E\cdot d\vec \ell = \int_{A\to B} dW$$

$$\frac{dW}{dt} = -\underbrace{q\,\vec E\cdot \overbrace{\frac{d\vec \ell}{dt}}^{\vec v}}_{\vec J}$$

$q \longrightarrow \rho$

$q\vec v \longrightarrow \vec J$

$$\frac{dW}{dt} = \int d^3x\, \vec E(\vec x)\cdot \vec J(\vec x)$$

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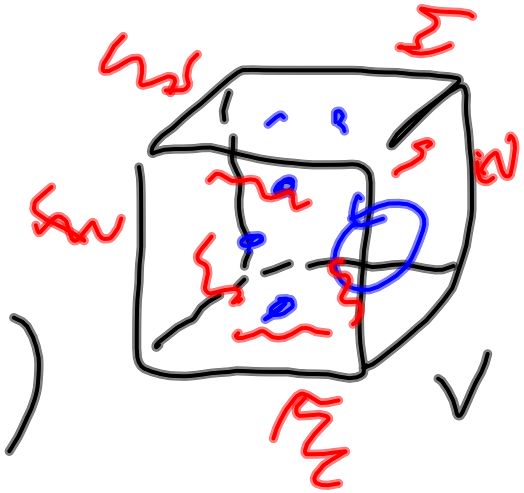
Work done by the fields on the charges

$\vec E\cdot \vec J$

$$\nabla\times\vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$



$$\int_V \vec{E}\cdot\vec{J}\, d^3x = \int_V d^3x\, \vec{E}\cdot\left(\nabla\times\vec{H} - \frac{\partial\vec{D}}{\partial t}\right)$$

$$\nabla\cdot(\vec{E}\times\vec{H}) = \vec{H}\cdot\underbrace{(\nabla\times\vec{E})}_{-\frac{\partial \vec{B}}{\partial t}} - \vec{E}\cdot(\nabla\times\vec{H})$$

$$\boxed{\int \vec{E}\cdot\vec{J}\, d^3x = \int_V d^3x\left[-\nabla\cdot(\vec{E}\times\vec{H}) - \vec{H}\cdot\frac{\partial\vec{B}}{\partial t} - \vec{E}\cdot\frac{\partial\vec{D}}{\partial t}\right]}$$