

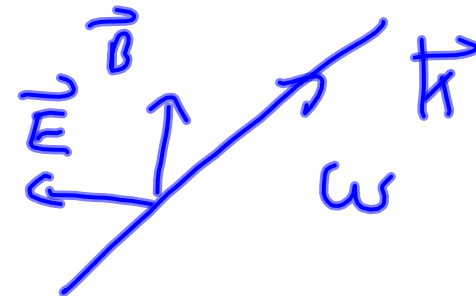
$\Delta W = \bar{F} \Delta x$ ← "smooth function"

$\frac{\Delta W}{\Delta t} = F \frac{\Delta x}{\Delta t} = q E v$

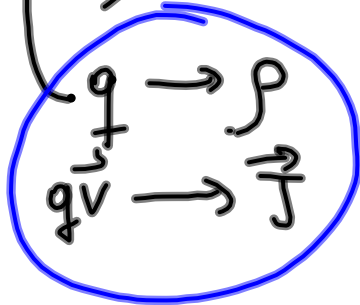
where $v = \frac{\Delta x}{\Delta t} = v(t)$ and $q E v$ is circled in red.

$\frac{dW}{dt} = F v(t)$

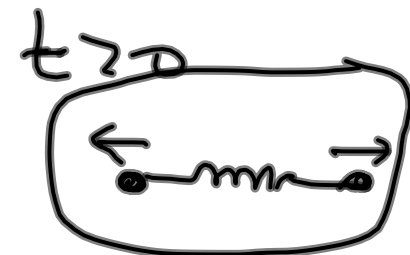
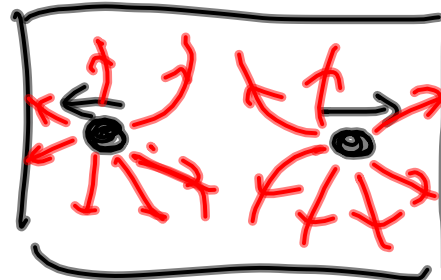
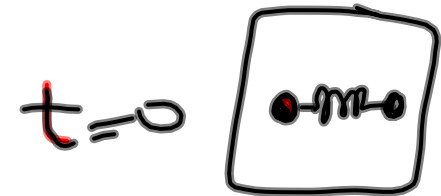
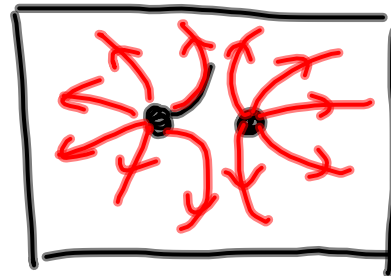
$\vec{E}(\vec{x}, t), \vec{B}(\vec{x}, t)$



$$\frac{dW}{dt} \propto \int d^3x \vec{E}(\vec{x}, t) \cdot \vec{J}(\vec{x}, t)$$



Conversion of E&M energy into mechanical energy.



$$\vec{E} \cdot \left[\nabla \times \vec{H} - \frac{\partial \vec{D}}{\partial t} \right] = \vec{J} \cdot \vec{E}$$

$$\vec{E} \cdot \nabla \times \vec{H} = \vec{H} \cdot \underbrace{(\nabla \times \vec{E})}_{-\frac{\partial \vec{B}}{\partial t}} - \nabla \cdot (\vec{E} \times \vec{H})$$

$$\vec{E} \cdot \frac{\partial \vec{D}}{\partial t} + \vec{H} \cdot \frac{\partial \vec{B}}{\partial t} = \vec{E} \cdot \frac{\partial \vec{E}}{\partial t} + \frac{1}{\mu} \vec{B} \cdot \frac{\partial \vec{B}}{\partial t}$$

$$\begin{aligned} \vec{D} &= \epsilon \vec{E} \\ \vec{B} &= \mu \vec{H} \end{aligned}$$

$$E = \frac{\epsilon}{2} \int \vec{E}^2 d^3x$$

$$\frac{\partial (\vec{E} \cdot \vec{E})}{\partial t} = 2 \vec{E} \cdot \frac{\partial \vec{E}}{\partial t}, \quad \frac{\partial (\vec{B} \cdot \vec{B})}{\partial t} = 2 \vec{B} \cdot \frac{\partial \vec{B}}{\partial t}$$

$$\int \vec{E} \cdot \vec{J} \, d^3x = \int d^3x \frac{\partial}{\partial t} \left[\frac{1}{2} (\epsilon \vec{E}^2 + \frac{1}{\mu} \vec{B}^2) \right] - \int d^3x \nabla \cdot (\vec{E} \times \vec{H})$$

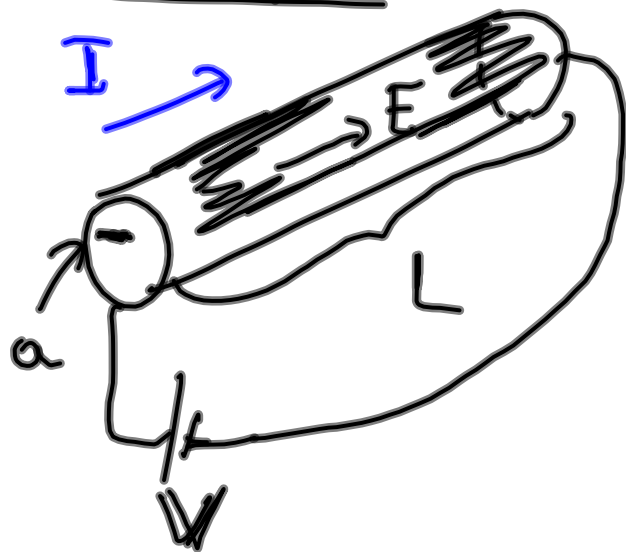
$\frac{\partial}{\partial t} \int \vec{E} \cdot \vec{J} \, d^3x$
 electromagnetic field

Pointing vector

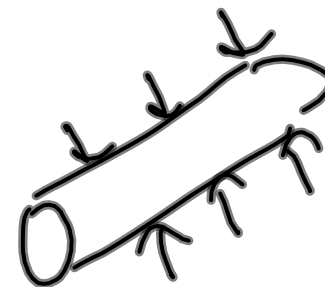
$$\int \nabla \cdot \vec{S} \, d^3x = \oint \vec{S} \cdot \vec{n} \, da$$

$$\frac{1}{\mu \epsilon} = c^2$$

Example



$$E = \frac{V}{L}$$



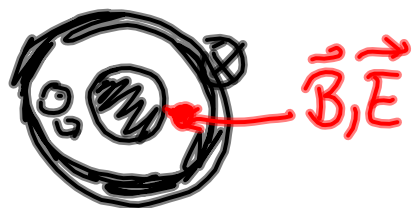
$$\oint_C \vec{B} \cdot d\vec{l} = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi a}$$



$$S = \frac{EB}{\mu_0} = \frac{V}{L} \frac{\mu_0 I}{\mu_0 2\pi a}$$

$$\int_S \vec{S} \cdot d\vec{a} = \frac{VI}{L} 2\pi a L$$



$$P = VI$$

Conservation of momentum

$$\vec{F} = \frac{d}{dt} \int_{\text{mechanical}} (\rho \vec{E} + \vec{J} \times \vec{B}) d^3x$$

$\vec{F} \xrightarrow{\frac{d}{dt}}$ $\int_{\text{mechanical}}$ $\rho \vec{E} \xrightarrow{\rho}$ $\vec{J} \times \vec{B} \xrightarrow{\vec{J}}$

Vacuum: $\nabla \cdot \vec{D} = \rho$, $\epsilon_0 \nabla \cdot \vec{E} = \rho$

$$\vec{J} = \nabla \times \vec{H} - \frac{\partial \vec{D}}{\partial t}$$

$\vec{H} = \frac{\vec{B}}{\mu_0}$

$$\rho \vec{E} + \vec{J} \times \vec{B} = \epsilon_0 \vec{E} (\nabla \cdot \vec{E}) + \frac{1}{\mu_0} (\nabla \times \vec{B}) \times \vec{B} - \epsilon_0 \underbrace{\frac{\partial \vec{E}}{\partial t} \times \vec{B}}$$

$$\vec{B} \times \frac{\partial \vec{E}}{\partial t} = \frac{\partial (\vec{B} \times \vec{E})}{\partial t} - \frac{\partial \vec{B}}{\partial t} \times \vec{E}$$