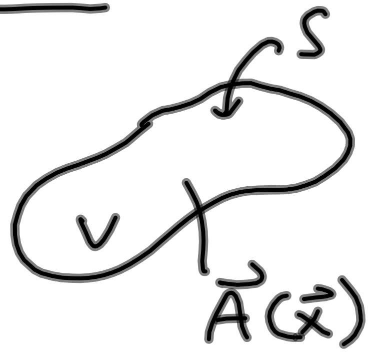
1.4 Diff. form of gauss's law

$$\oint_S (\vec{A}\cdot\vec{n})\,dS = \int_V (\nabla\cdot\vec{A})\,dV$$



$$\oint_S (\vec{E}\cdot\vec{n})\,dS = \frac{1}{\epsilon_0}\int_V \rho(\vec{x})\,d^3x = \int_V (\nabla\cdot\vec{E})\,d^3x$$

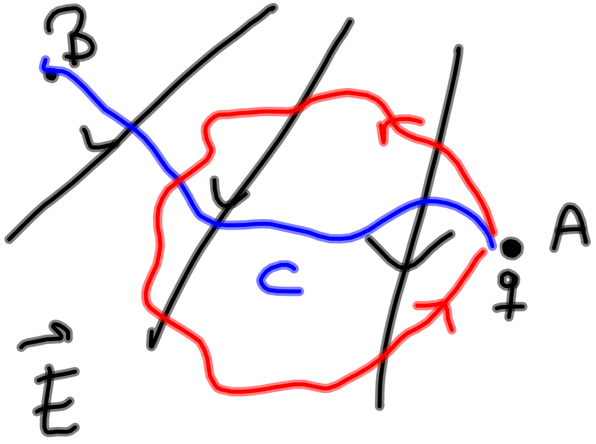
$$\boxed{\nabla\cdot\vec{E} = \frac{\rho(\vec{x})}{\epsilon_0}}$$

1.5 Scalar potential

$$\vec{E}(\vec{x}) = \frac{1}{4\pi\epsilon_0}\int_V \rho(\vec{x}')\underbrace{\frac{(\vec{x}-\vec{x}')}{|\vec{x}-\vec{x}'|^3}}_{-\nabla_x\left(\frac{1}{|\vec{x}-\vec{x}'|}\right)} d^3x'$$

$$= -\nabla \underbrace{\frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{x}')}{|\vec{x}-\vec{x}'|} d^3x'}_{\Phi(\vec{x})}$$

$$\boxed{\nabla\times\vec{E} = -\nabla\times\nabla\Phi = 0}\ , \quad \boxed{\nabla\cdot\vec{E} = \frac{\rho(\vec{x})}{\epsilon_0}}$$

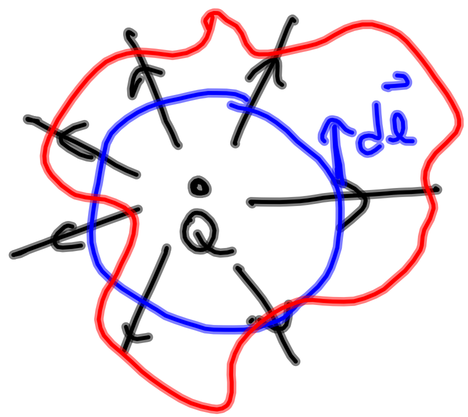


Work $A \to B$

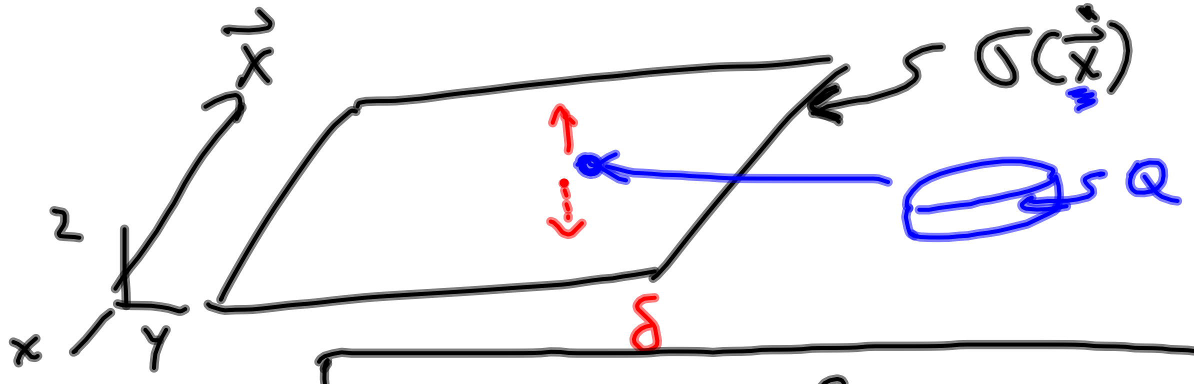
$\vec{F} = q\vec{E}$

$$W = \underbrace{q(\Phi_B - \Phi_A)}_{\text{indep. of } C}$$

$$\oint_C \vec{E} \cdot \vec{dl} = 0$$

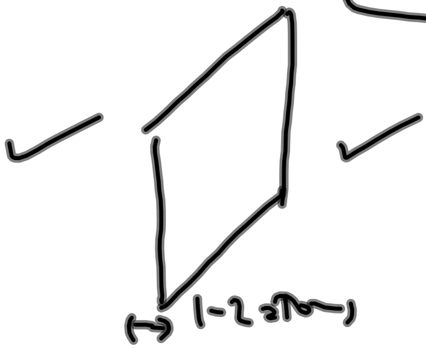


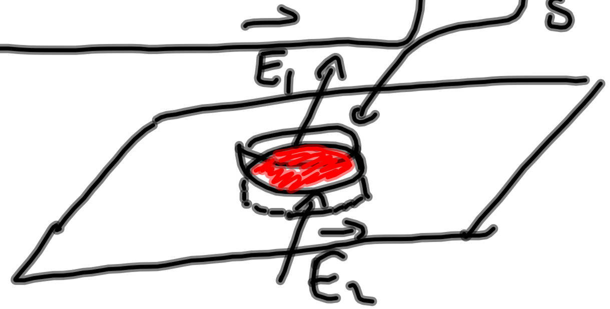
1.6 Surface distribution of charges



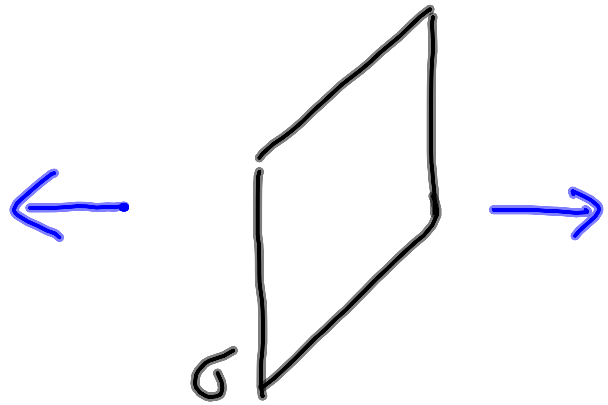
$$\boxed{\Phi(\vec{x}) = \frac{1}{4\pi\epsilon_0}\int_S \frac{\sigma(\vec{x}')}{|\vec{x}-\vec{x}'|}\,ds'}$$

$\oint_S (\vec{E}\cdot\vec{n})\,ds$



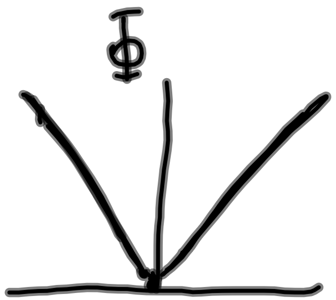


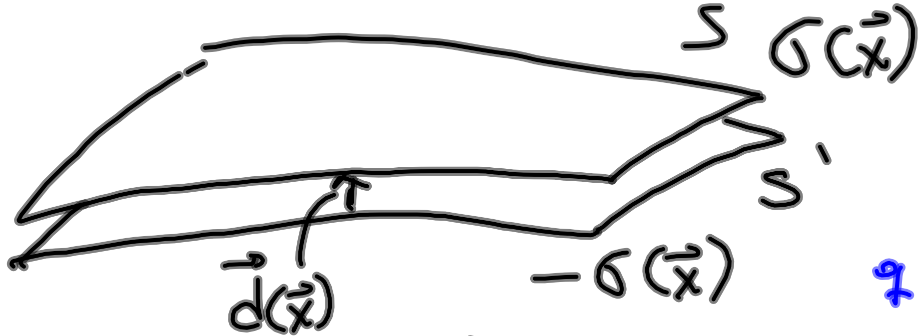
$$\boxed{\left(\vec{E}_2(\vec{x}) - \vec{E}_1(\vec{x})\right)\cdot\vec{n} = \frac{\sigma(\vec{x})}{\epsilon_0}}$$



$\vec{E}$ is discontinuous

Φ is continuous





"dipole layer"

q, $-q$, a

$a \to 0$

$q \to \infty$

$$|\vec{d}| \to 0$$

$$\sigma \to \infty$$

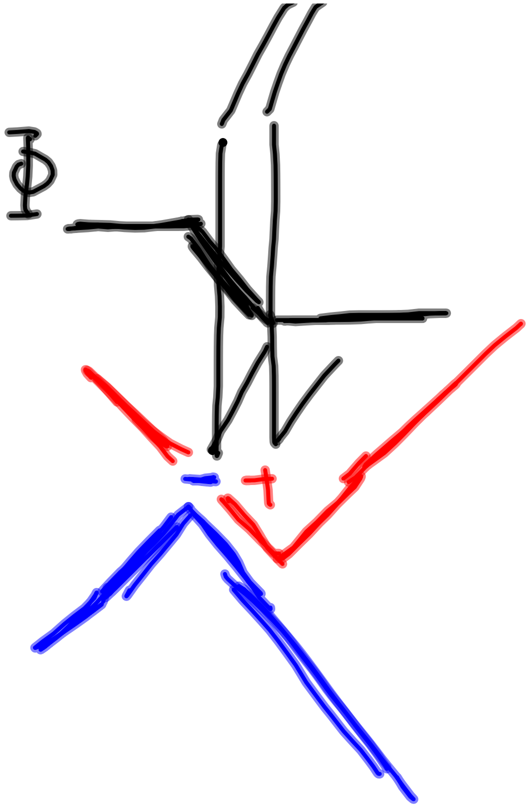
$$\underbrace{\sigma}_{\to\infty} \cdot \underbrace{d}_{\to 0} = D(\vec{x})$$

$$\Phi = \Phi_S + \Phi_{S'}$$

$$\Phi_S = \frac{1}{4\pi\epsilon_0}\int \frac{\sigma(\vec{x}')}{|\vec{x}-\vec{x}'|}\, d^2x'$$

$$\Phi_{S'} = -\frac{1}{4\pi\epsilon_0}\int \frac{\sigma(\vec{x}')\, d^2x'}{|\vec{x}-\vec{x}'+\vec{n}\, d(\vec{x}')|} = \frac{1}{4\pi\epsilon_0}\int_S \ldots d^2x'$$

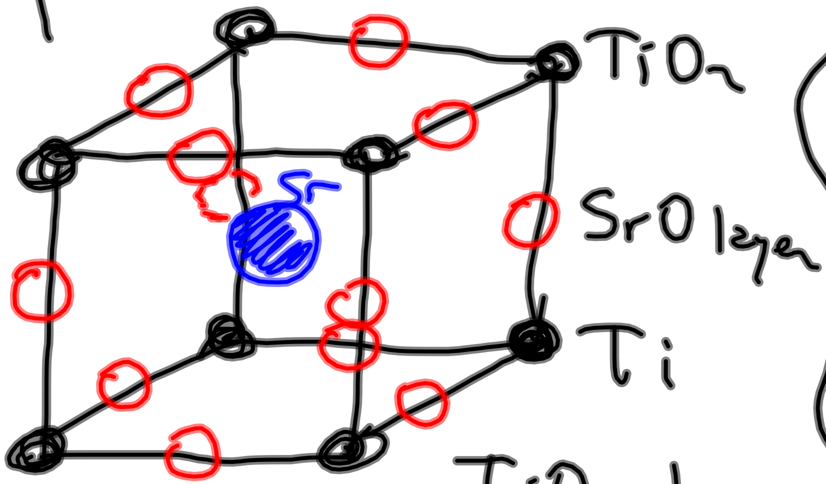
$$\ldots = D(\vec{x}')\, \vec{n} \cdot \nabla' \left(\frac{1}{|\vec{x}-\vec{x}'|}\right)$$



Φ is discontinuous

$LaAlO_3 / SrTiO_3$

perovskite structure



Sr^{++}

$O^{--} \times 3 = 6-$

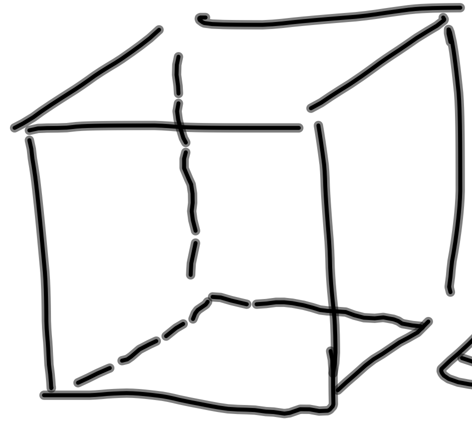
Ti^{++++}

$\underbrace{TiO_2}$ layer

$Ti^{++++}O_2^{--}$ neutral

$LaAlO_3$

+++ +++ --



$\leftarrow AlO_2$ $-$

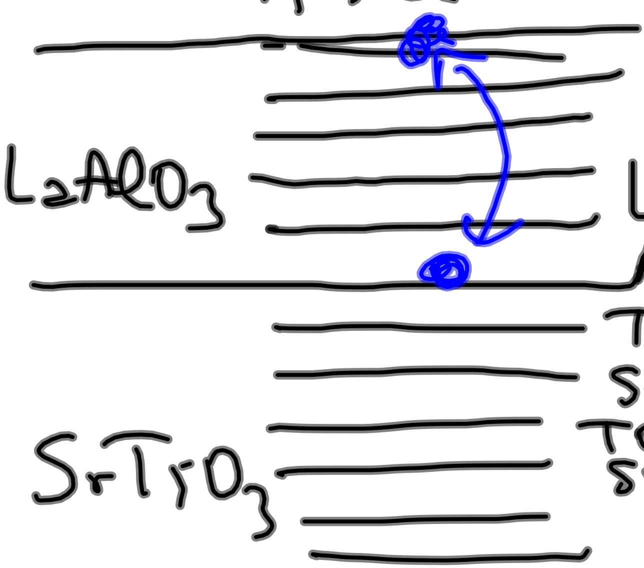
+++ --

$\leftarrow LaO$ $+$

+++ --

$\leftarrow AlO_2$ $-$

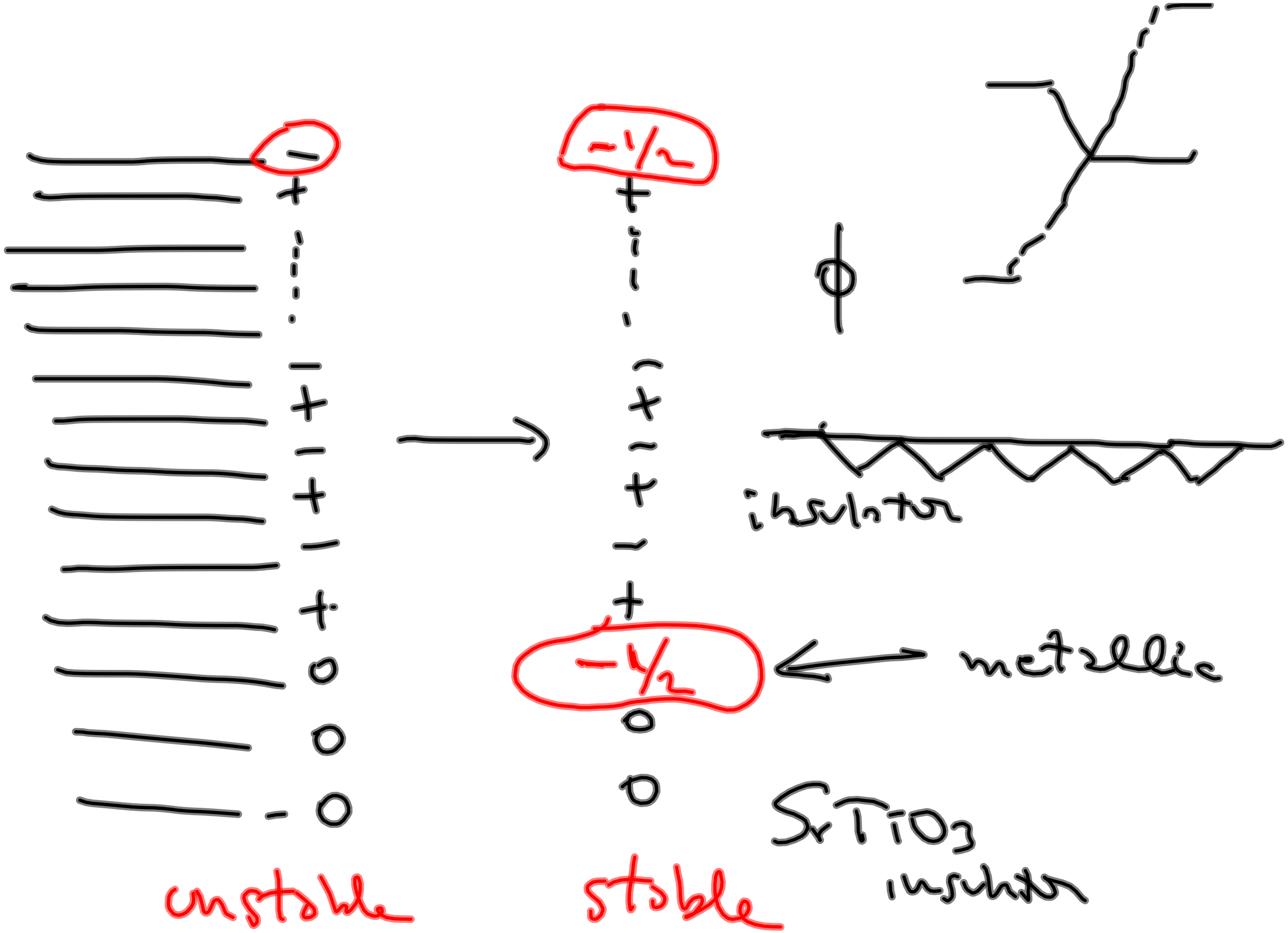
Surface



$\pm$ $\pm$ $\pm$ $\pm$ 0 0 0 0

"polar catastrophe"

Φ


-1/2
-1/2
metallic
insulator
SrTiO3
insulator
unstable
stable