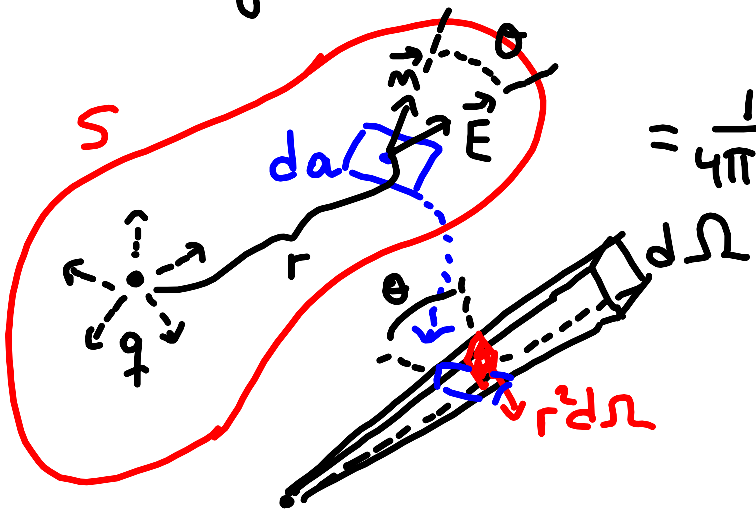
1.3 Gauss's law



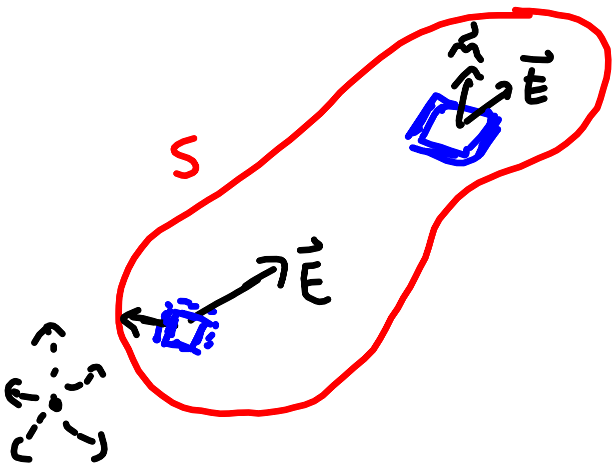
$$(\vec{E}\cdot\vec{n})\,da = |\vec{E}||\vec{n}|\cos\theta\,da =$$

geometry

$$= \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\cdot 1\cdot\underbrace{\cos\theta\,da}_{r^2 d\Omega} = \frac{q}{4\pi\epsilon_0}\frac{1}{\cancel{r^2}}\cancel{r^2}\,d\Omega$$

physics

$$\oint_S(\vec{E}\cdot\vec{n})\,da = \frac{q}{4\pi\epsilon_0}\underbrace{\oint_S d\Omega}_{4\pi} = \frac{q}{\epsilon_0}\quad\left(\text{if } q \text{ inside}\right)$$

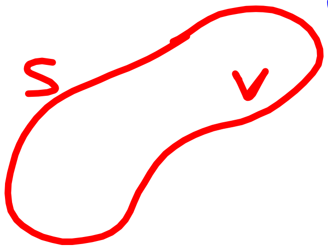


$$\oint_S(\vec{E}\cdot\vec{n})\,da = 0\quad(\text{if } q \text{ outside})$$

- Generalized to $\sum_i q_i$ (inside S)
- Generalized to $\int\rho(\vec{x}')\,d^3x'$

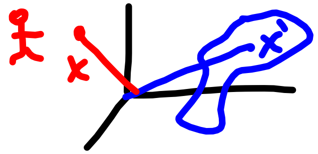
1.4 Differential form of Gauss's law

$$\oint_S (\vec{E}\cdot\vec{n})\,da \underset{\uparrow \text{ physics}}{=} \frac{1}{\epsilon_0}\int_V \rho(\vec{x}')\,d^3x' \underset{\uparrow \text{ divergence theorem}}{=} \int_V (\nabla\cdot\vec{E})\,d^3x'$$



$$\boxed{\nabla\cdot\vec{E}(\vec{x}) = \frac{\rho(\vec{x})}{\epsilon_0}}$$

1.5 Scalar Potential



$$\vec{E}(\vec{x}) = \frac{1}{4\pi\epsilon_0}\int_V \rho(\vec{x}')\,\frac{(\vec{x}-\vec{x}')}{|\vec{x}-\vec{x}'|^3}\, d^3x' =$$

last lecture

V ← all charges

check

$$\frac{(\vec{x}-\vec{x}')}{|\vec{x}-\vec{x}'|^3} \doteq -\nabla_{\vec{x}}\left(\frac{1}{|\vec{x}-\vec{x}'|}\right)$$

Use $|\vec{u}| = \sqrt{\vec{u}\cdot\vec{u}} = \sqrt{u_x^2+u_y^2+u_z^2}$

$u_x = x-x'$

$u_y = y-y'$

$u_z = z-z'$

$$= -\nabla_{\vec{x}}\, \underbrace{\frac{1}{4\pi\epsilon_0}\int_V \frac{\rho(\vec{x}')\,d^3x'}{|\vec{x}-\vec{x}'|}}_{\equiv\, \Phi(\vec{x})}$$

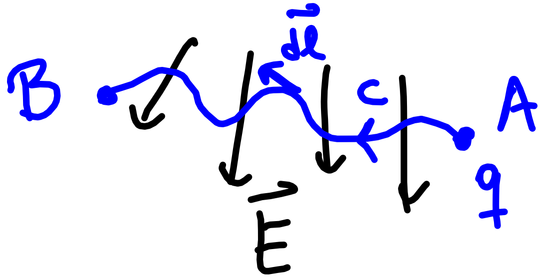
$$\boxed{\vec{E}(\vec{x}) = -\nabla_{\vec{x}}\,\Phi(\vec{x})}$$

check

$$\nabla\times\vec{E} = -\nabla\times\nabla\Phi \doteq 0\,; \quad \boxed{\nabla\times\vec{E}(\vec{x}) = 0}$$

curl

Physical interpretation of Φ



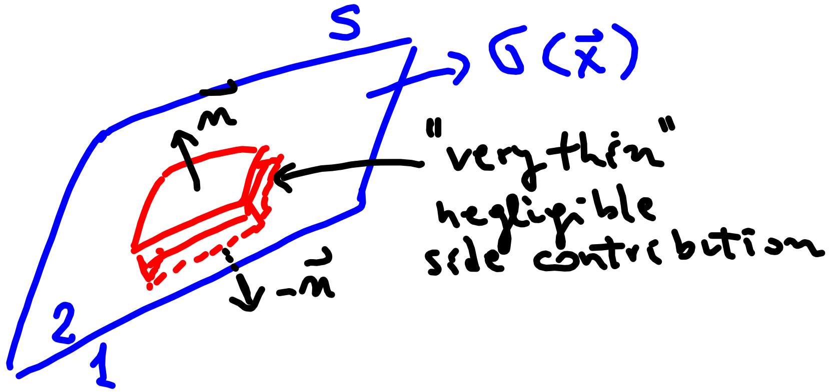
$$W_{A\to B} = -\int_A^B \vec{Force}\cdot\vec{dl} =$$

$$= -q\int_A^B \underbrace{\vec{E}(\vec{x})}_{-\nabla\Phi}\cdot\vec{dl} = q\int_A^B \nabla\Phi\cdot\vec{dl} \overset{\text{check}}{=}$$

$$= q\int_A^B d\Phi = \underbrace{q(\Phi_B - \Phi_A)}_{\text{independent of shape of C}} \quad ; \quad \boxed{q\Phi = \text{potential energy}}$$

Special cases: $\boxed{\oint_C \vec{E}\cdot\vec{dl} = 0}$

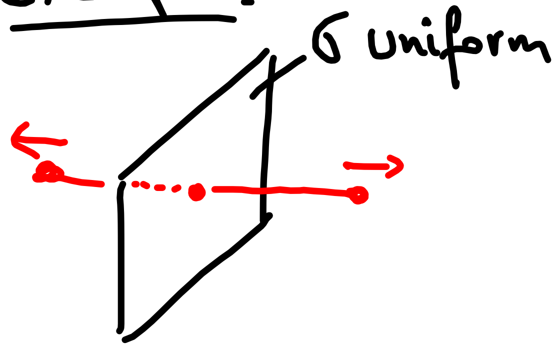
1.6 Surface distribution of charge



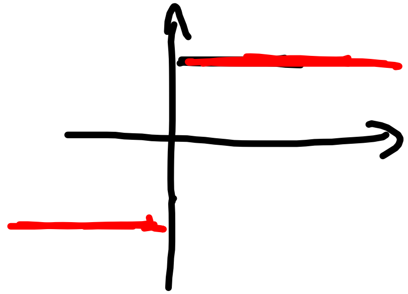
$$\oint_S (\vec{E}\cdot\vec{n})\,da = \frac{1}{\epsilon_0}\int_S \sigma(\vec{x})\,da$$

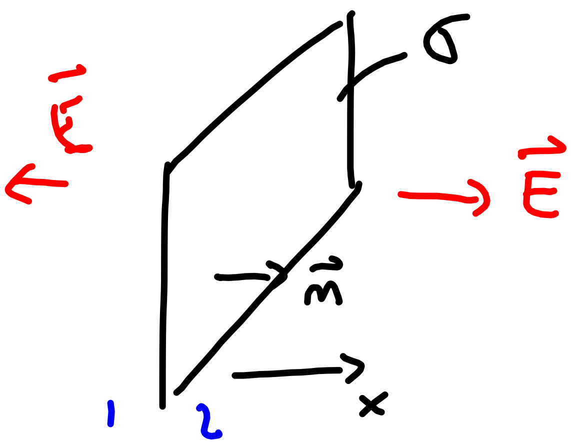
$$\boxed{\left(\vec{E}_2(\vec{x})-\vec{E}_1(\vec{x})\right)\cdot\vec{n} = \frac{\sigma(\vec{x})}{\epsilon_0}}$$

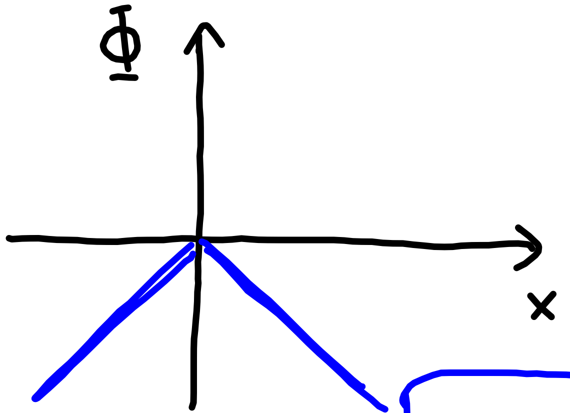
Example:



$\vec{E}$ is discontinuous







$$\boxed{\Phi = -\frac{\sigma}{2\epsilon_0}|x|}$$

$$-\nabla\Phi = E = \frac{\sigma}{2\epsilon_0}$$

$$(\vec E_2 - \vec E_1)\cdot\vec n = 2E = \frac{\sigma}{\epsilon_0}$$

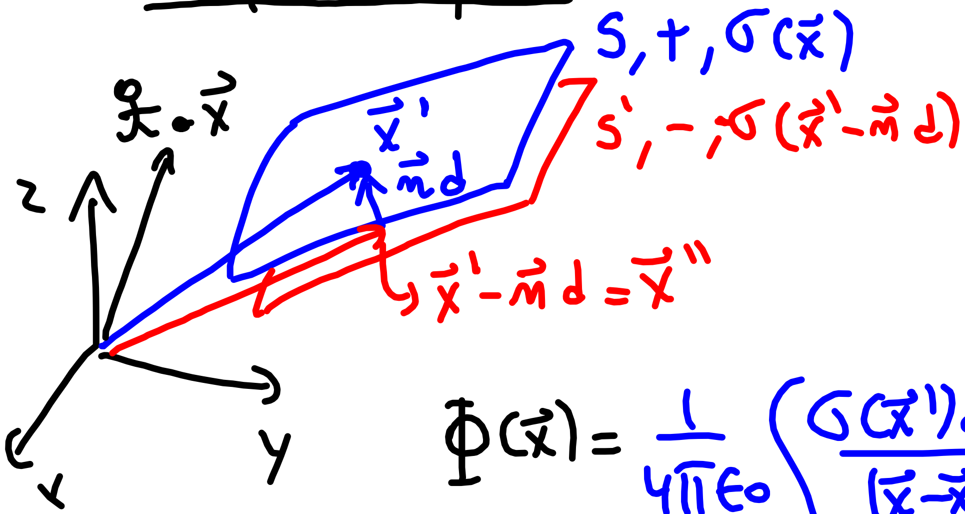
$$\vec E_2 = -\vec E_1$$

$$\boxed{E = \frac{\sigma}{2\epsilon_0}}$$

Φ is continuous

Dipole layers

$\Phi(\vec{x})$?



$$\Phi(\vec{x}) = \frac{1}{4\pi\epsilon_0}\int_S \frac{\sigma(\vec{x}')da'}{|\vec{x}-\vec{x}'|} - \frac{1}{4\pi\epsilon_0}\int \frac{\sigma(\vec{x}')da'}{|\vec{x}-\vec{x}'+\vec{n}d|}$$

check

$$\frac{1}{|\vec{n}+\vec{\epsilon}|} \approx \frac{1}{|\vec{n}|} + \vec{\epsilon}\cdot\nabla\left(\frac{1}{|\vec{n}|}\right)$$

$$\nabla_{\vec{x}} = -\nabla_{\vec{x}'}$$

$$\boxed{\Phi(\vec{x}) = \frac{1}{4\pi\epsilon_0}\int_S \sigma(\vec{x}')d(\vec{x}')\,\vec{n}\cdot\nabla'\left(\frac{1}{|\vec{x}-\vec{x}'|}\right)da'}$$