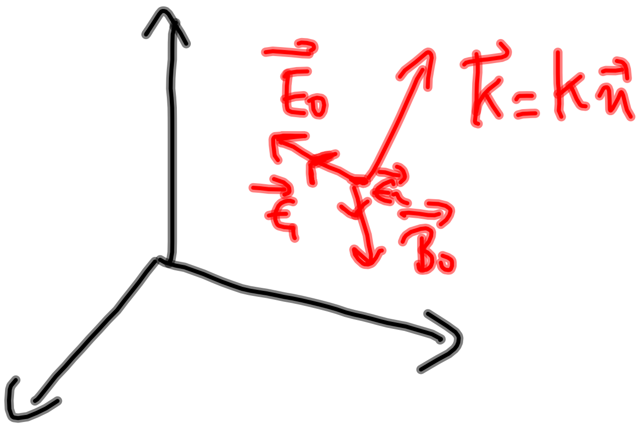


$$\vec{S} = \frac{1}{2}(\vec{E} \times \vec{H}^*)$$

f, g

$$\langle fg \rangle \underset{\text{def}}{=} \frac{1}{\sim T}\int_0^{\sim T} dt \; f g = \frac{ab}{2}$$

$$T = \frac{2\pi}{\omega}$$

$$f = a \overset{e}{\cos}(kx - \omega t)$$
$$g = b \underset{e^{i(kx-\omega t)}}{\cos}(kx - \omega t)$$

$$\cos^2\varphi = \frac{1}{2} + \frac{1}{2}\cos(2\varphi), \quad \int_0^T dt \cos[2(kx - \omega t)] = 0$$

$$\langle fg \rangle = \frac{fg^*}{2}$$

$$\vec{B}_0 = \frac{n}{c}(\vec{n}\times\vec{E}_0)$$

$$\boxed{n = \sqrt{\frac{\mu\varepsilon}{\mu_0\varepsilon_0}}}$$

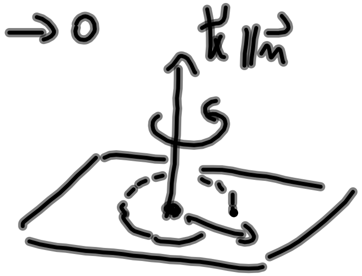
$$\vec{S} = \frac{1}{2}(\vec{E}\times\vec{H}^*) = \frac{1}{2}\sqrt{\frac{\varepsilon}{\mu}}\,|\vec{E}_0|^2\,\vec{n}$$

time average

$$\text{Energy density} = u = \frac{1}{4}\left(\epsilon\,\vec{E}\cdot\vec{E}^* + \frac{1}{\mu}\vec{B}\cdot\vec{B}^*\right) = \frac{\varepsilon}{2}|\vec{E}_0|^2$$

$$\frac{1}{T}\int_0^T dt\,\underbrace{\left(\frac{\varepsilon}{2}\vec{E}\cdot\vec{E}+\cdots\right)}_{\text{old } u}$$

$\omega \to 0$ $\quad \vec{k} \parallel \vec{n}$



linear or circular polarization

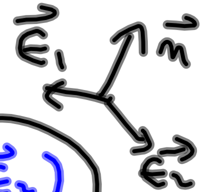
$$\vec{E}_1 = \vec{\epsilon}_1 E_1 e^{i(\vec{k}\cdot\vec{x}-\omega t)}$$

$$\vec{E}_2 = \vec{\epsilon}_2 E_2 e^{i(\vec{k}\cdot\vec{x}-\omega t)}$$

$$\underbrace{e^{i\varphi}}_{i} \quad \pi/2$$

if $E_1 = E_2$

$$(\vec{\epsilon}_1 + i\,\vec{\epsilon}_2)$$



$$\vec{E} = (\vec{\epsilon}_1 E_1 + \vec{\epsilon}_2 E_2)\, e^{i(\vec{k}\cdot\vec{x} - \omega t)} = (\vec{\epsilon}_1 + i\,\vec{\epsilon}_2)\, E_1\, e^{i(\vec{k}\cdot\vec{x} - \omega t)}$$

i

$e^{i\pi/2} = i$

$E_1 = E_2$

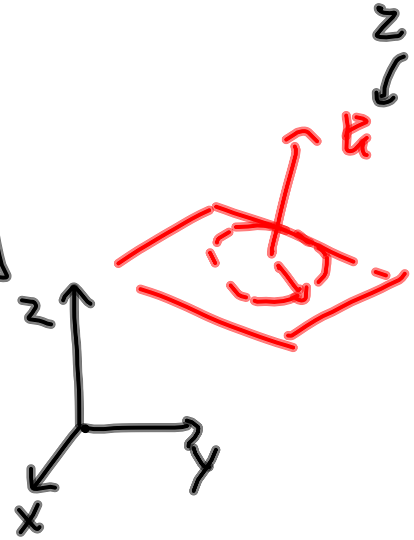
$$\mathrm{Re}\,\vec{E}(\vec{x},t) = \mathrm{Re}\left[(\vec{\epsilon}_1 + i\,\vec{\epsilon}_2)\, E_1\, e^{i(\vec{k}\cdot\vec{x} - \omega t)}\right]$$

along $\vec{\epsilon}_1$

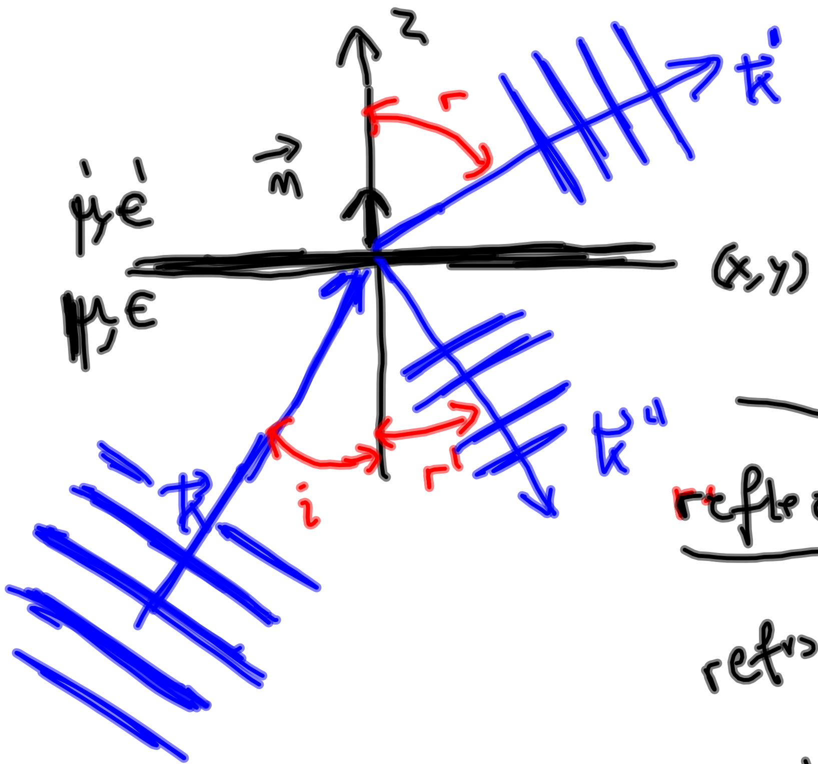
$$\mathrm{Re}\,E_x = E_1 \cos(\vec{k}\cdot\vec{x} - \omega t)$$

$$\mathrm{Re}\,E_y = -E_1 \sin(\vec{k}\cdot\vec{x} - \omega t)$$

along $\vec{\epsilon}_2$



7.3 Reflection and Refraction of Plane Waves



refracted

given

$$\vec E = \vec E_0 \, e^{i(\vec k \cdot \vec x - \omega t)} \qquad \text{incident}$$

$$\vec B_0 = \sqrt{\mu \varepsilon}\left(\frac{\vec k \times \vec E_0}{k}\right)$$

refracted

$$\vec E' = \vec E_0' \, e^{i \vec k' \cdot \vec x} \, e^{-i\omega t}$$

$$\vec B_0' = \sqrt{\mu' \varepsilon'}\left(\frac{\vec k' \times \vec E_0'}{k'}\right)$$

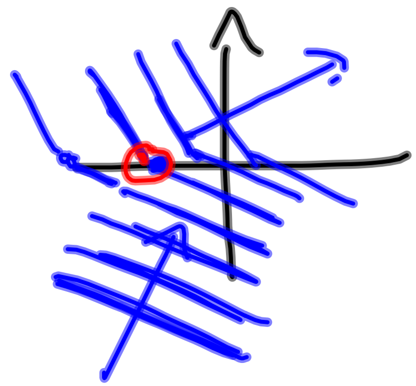
reflected

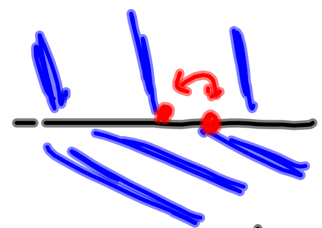
$$\vec E'' = \vec E_0'' \, e^{i \vec k'' \cdot \vec x} \, e^{-i\omega t}$$

$$\vec B_0'' = \sqrt{\mu \varepsilon}\left(\frac{\vec k'' \times \vec E_0''}{k''}\right)$$

k

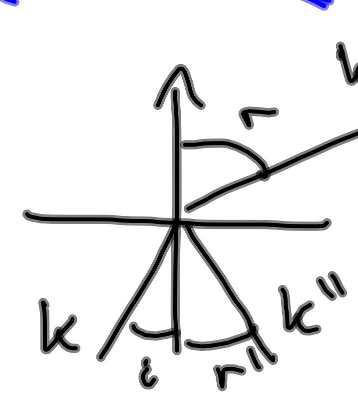
Kinematic properties





$$\vec{k}\cdot\vec{x}\Big|_{z=0} = \vec{k}'\cdot\vec{x}\Big|_{z=0} = \vec{k}''\cdot\vec{x}\Big|_{z=0}$$

$$k\sin(i)x = k'\sin(r)x \qquad k''\sin(r')x$$



$k'' = k$ $\qquad \boxed{i = r'}$

$$\boxed{\frac{\sin(i)}{\sin(r)} = \frac{k'}{k} = \frac{\cancel{\omega}\sqrt{\mu'\varepsilon'}}{\cancel{\omega}\sqrt{\mu\varepsilon}} = \frac{n'}{n}}$$

Snell's laws

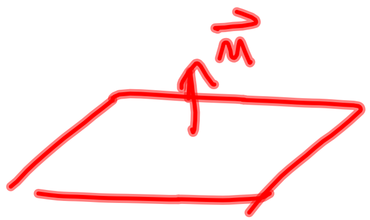
Boundary conditions:

(linear medium)

Normal components of $\vec{D}$ and $\vec{B}$ continuous

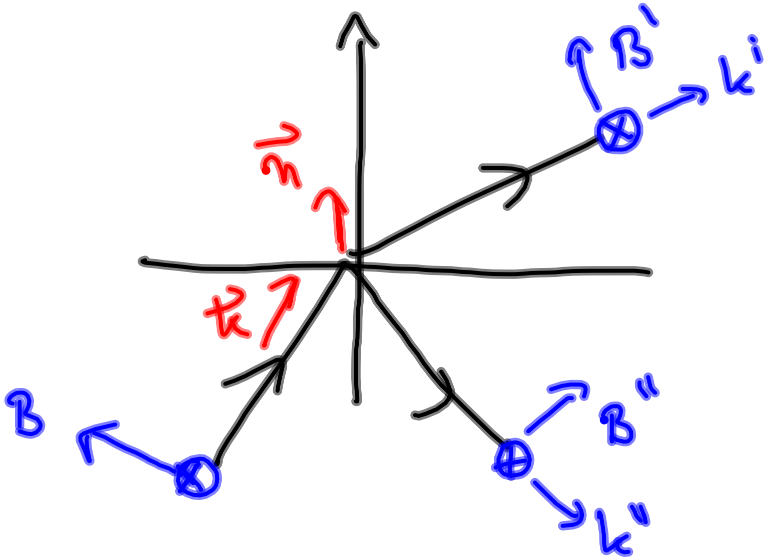
Tangential components of $\vec{E}$ and $\vec{H}$ are continuous

$$\left\{\begin{aligned}
&\Big[\underbrace{\epsilon(\vec{E}_0+\vec{E}_0'')}_{\vec{D}_i+\vec{D}_{reflected}} - \underbrace{\epsilon'\vec{E}_0'}_{\vec{D}_{refracted}}\Big]\cdot\vec{n} = 0\\
&\left(\vec{k}\times\vec{E}_0 + \vec{k}''\times\vec{E}_0'' - \vec{k}'\times\vec{E}_0'\right)\cdot\vec{n} = 0\\
&\left(\vec{E}_0+\vec{E}_0''-\vec{E}_0'\right)\times\vec{n} = 0\\
&\left[\frac{\vec{k}\times\vec{E}_0+\vec{k}''\times\vec{E}_0''}{\mu} - \frac{\vec{k}'\times\vec{E}_0'}{\mu'}\right]\times\vec{n} = 0
\end{aligned}\right.$$



$|\vec{k}''| = |\vec{k}|$

$k'' = k$



$\perp$ to the plane of incidence