Today: return Sec. 6.3, then 6.4, then Ch. 9

6.3 Reminder In Lorentz gauge,

$$\nabla^2\phi - \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} = -\rho/\epsilon_0 \quad \rho(\vec{x},t)$$

$$\nabla^2\vec{A} - \frac{1}{c^2}\frac{\partial^2\vec{A}}{\partial t^2} = -\mu_0\vec{J} \quad \vec{J}(\vec{x},t)$$

equiv. to the 4 Max. Eqs.

Solutions?

$$\boxed{\nabla^2\psi(\vec{x},t) - \frac{1}{c^2}\frac{\partial^2\psi(\vec{x},t)}{\partial t^2} = -4\pi f(\vec{x},t)}$$

Propose: $\underbrace{\psi(\vec{x},t)}_{\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}} = \int d^3x' \int dt' \underbrace{G(\vec{x},t;\vec{x}',t')}_{\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}} f(\vec{x}',t') = -4\pi f(\vec{x},t)$

$$\left[\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right] G(\vec{x},t;\vec{x}',t') = -4\pi\,\delta(\vec{x}-\vec{x}')\,\delta(t-t')$$

Propose:

$$G(\vec{x},t;\vec{x}',t') = \frac{1}{\underbrace{|\vec{x}-\vec{x}'|}_{R}}\frac{1}{2\pi}\int_{-\infty}^{+\infty} d\omega\, e^{i\frac{\omega}{c}\overbrace{|\vec{x}-\vec{x}'|}^{R}}\, e^{-i\omega(t-t')}$$

$$\left[\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right]\frac{1}{R}\frac{1}{2\pi}\int d\omega\, e^{i\frac{\omega}{c}R}\, e^{-i\omega(t-t')} =$$

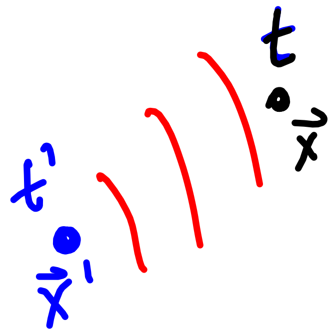
$$= \frac{1}{2\pi}\int d\omega\, e^{-i\omega(t-t')}\,\nabla^2\left(\frac{e^{\frac{i\omega R}{c}}}{R}\right) - \frac{1}{c^2}\frac{1}{2\pi}\int d\omega\,\frac{e^{\frac{i\omega R}{c}}}{R}\,\underbrace{\frac{\partial^2}{\partial t^2}e^{-i\omega(t-t')}}_{-\omega^2 e^{-i\omega(t-t')}}$$

$$\nabla^2\left(\frac{1}{|\vec{x}-\vec{x}'|}\right) = -4\pi\,\delta(\vec{x}-\vec{x}')$$

$R \rightarrow R+a$; and $a \rightarrow 0$

$R \neq 0$; easy $\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right) G = 0$

$\left[R \to R + a;\ a \to 0 \qquad \nabla^2\left(\frac{1}{R}\right) = -4\pi\,\delta(R) \right]$



$$\frac{1}{2\pi}\int_{-\infty}^{+\infty} d\omega\, e^{i\omega x} = \delta(x)$$

$$G(\vec{x}, t; \vec{x}', t') = \frac{1}{|\vec{x}-\vec{x}'|}\underbrace{\frac{1}{2\pi}\int d\omega\, e^{i\omega\frac{|\vec{x}-\vec{x}'|}{c}}\, e^{-i\omega(t-t')}}_{\delta\left(t' - t + \frac{|\vec{x}-\vec{x}'|}{c}\right)}$$

Retarded Green function

$$\boxed{t = t' + \frac{|\vec{x}-\vec{x}'|}{c}}$$

causality

$$\psi(\vec{x},t) = \int d^3x' \int dt' \underbrace{\frac{1}{|\vec{x}-\vec{x}'|}\,\delta\!\left(t-t'-\frac{|\vec{x}-\vec{x}'|}{c}\right)}_{G} f(\vec{x}',t') =$$

$$= \int d^3x' \frac{1}{|\vec{x}-\vec{x}'|} \underbrace{\int dt'\, \delta\!\left(t-t'-\frac{|\vec{x}-\vec{x}'|}{c}\right) f(\vec{x}',t')}_{f\left(\vec{x}',\, t-\frac{|\vec{x}-\vec{x}'|}{c}\right)}$$

$$\psi = \int d^3x' \frac{f(\vec{x}',t')_{ret}}{|\vec{x}-\vec{x}'|}$$

reminder $t' = t - \frac{|\vec{x}-\vec{x}'|}{c}$

$$\boxed{\begin{aligned} \phi(\vec{x},t) &= \frac{1}{4\pi\epsilon_0}\int d^3x' \, \frac{\rho(\vec{x}',t')_{ret}}{|\vec{x}-\vec{x}'|} \\ \vec{A}(\vec{x},t) &= \frac{\mu_0}{4\pi}\int d^3x' \, \frac{\vec{J}(\vec{x}',t')_{ret}}{|\vec{x}-\vec{x}'|} \end{aligned}}$$

Ch. 9

One frequency ω

$$\boxed{\begin{aligned}\rho(\vec{x}',t') &= \rho(\vec{x}')e^{-i\omega t'}\\ \vec{J}(\vec{x}',t') &= \vec{J}(\vec{x}')e^{-i\omega t'}\end{aligned}}$$

$$\underbrace{\vec{A}(\vec{x},t)}_{\vec{A}(\vec{x})\cancel{e^{-i\omega t}}} = \frac{\mu_0}{4\pi}\int d^3x' \frac{\vec{J}(\vec{x}')e^{-i\omega t}}{|\vec{x}-\vec{x}'|} \rightarrow e^{-i\omega(\cancel{t}-\frac{|\vec{x}-\vec{x}'|}{c})}$$

$$\boxed{\vec{A}(\vec{x}) = \frac{\mu_0}{4\pi}\int d^3x' \frac{\vec{J}(\vec{x}')}{|\vec{x}-\vec{x}'|} e^{i\omega\frac{|\vec{x}-\vec{x}'|}{c}}}$$

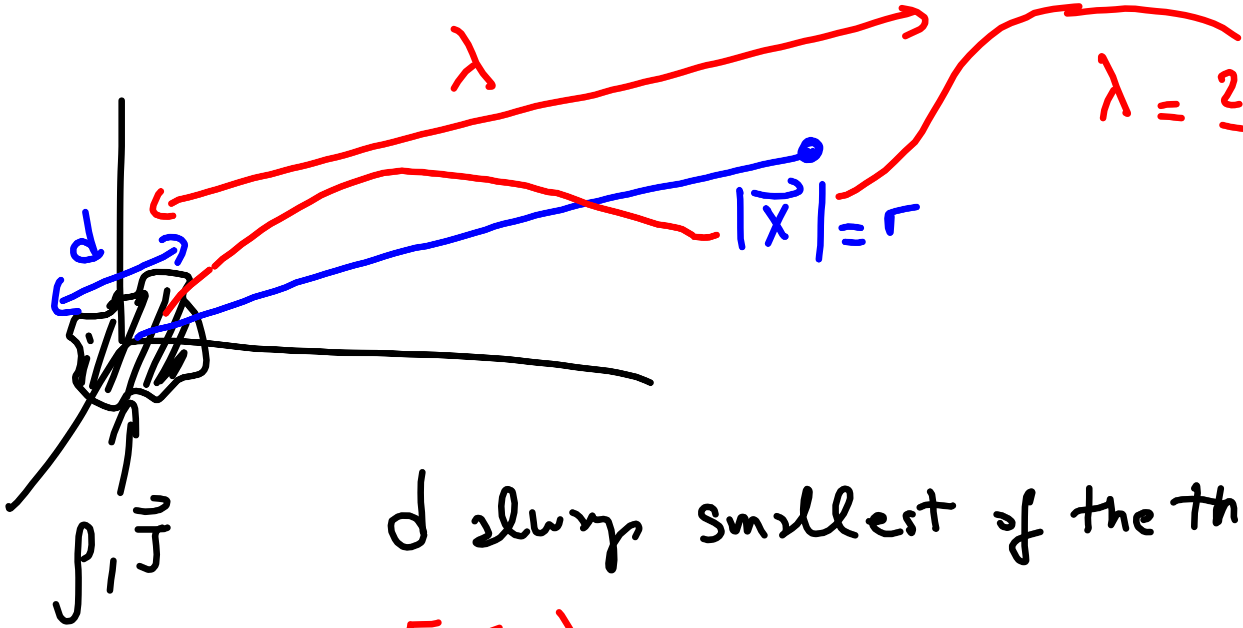
if $\omega = 0$ back to Ch. 5

$$\vec{A}(\vec{x}) \longrightarrow \boxed{\vec{B}(\vec{x}) = \nabla \times \vec{A}(\vec{x})}$$

outside the sources $\vec{J} = 0$, $\nabla \times \vec{H} = \cancel{\vec{J}} + \dfrac{\partial \overbrace{(\epsilon_0 \vec{E})}^{\vec{D}}}{\partial t}$

$$\boxed{\vec{E}(\vec{x}) = \frac{i}{k}\sqrt{\frac{\mu_0}{\epsilon_0}}\left[\nabla \times \vec{H}(\vec{x})\right]} \qquad \left(k = \frac{\omega}{c}\right)$$

Three length scales for $\vec{A}$: d, r, λ.



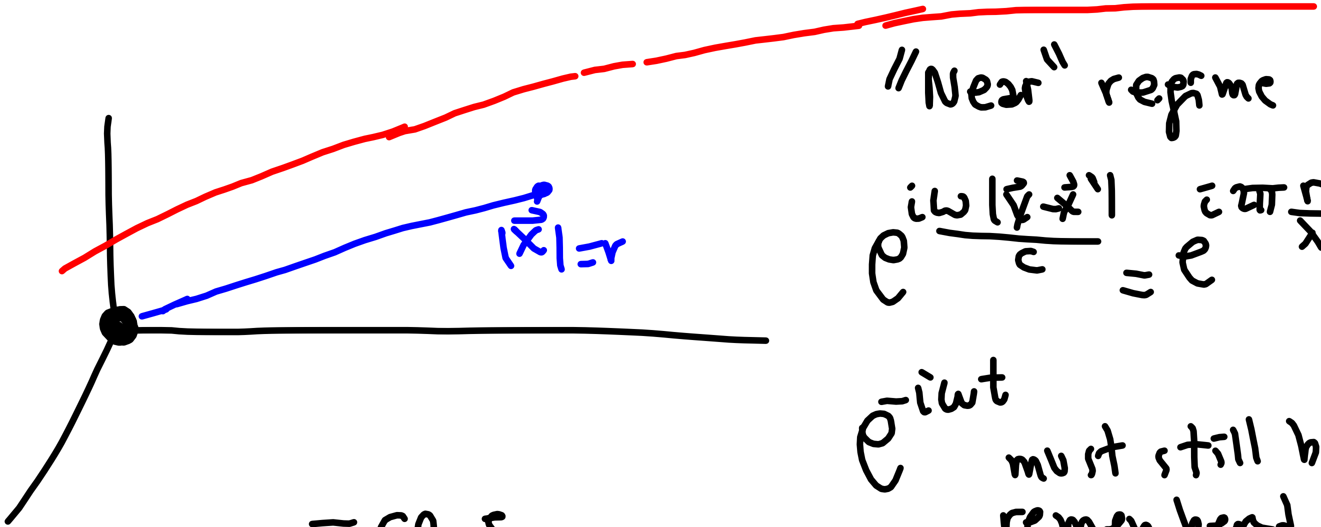
$$\lambda = \frac{2\pi c}{\omega}$$

d always smallest of the three

$r \ll \lambda$

$r \sim \lambda$

$r \gg \lambda$



"Near" regime

$$e^{\frac{i\omega|\vec{x}-\vec{x}'|}{c}} = e^{i 2\pi \frac{r}{\lambda}} \approx 1$$

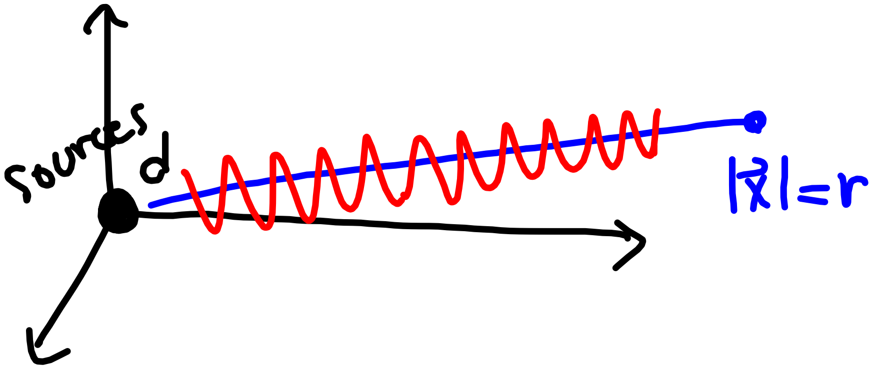
$e^{-i\omega t}$ must still be remembered

$\equiv$ Ch. 5

$$\underbrace{e^{-i\omega t}\vec{A}(\vec{x})}_{\vec{A}(\vec{x},t)} = \frac{\mu_0}{4\pi}\int d^3x' \, \frac{\vec{J}(\vec{x}')}{|\vec{x}-\vec{x}'|} \cdot e^{-i\omega t}$$

$\boxed{\text{The same as in Ch. 5}}$

$r \gg \lambda$ "Far"



Next lecture