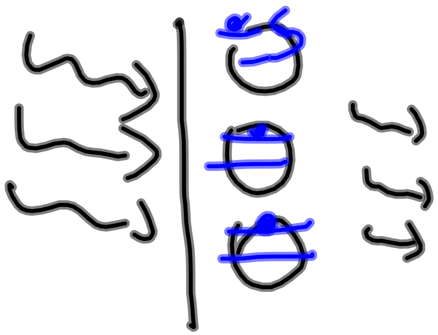
$$e^{i\vec{k}\cdot\vec{r}} \qquad \frac{\omega}{k} = \frac{c}{n} = \frac{1}{\sqrt{\mu\varepsilon}}$$

ε Re + i Im

$$e^{-\alpha x}, \quad \alpha = \frac{\omega^2}{c^2\,\mathrm{Re}\,k}\,\mathrm{Im}\left(\frac{\epsilon}{\epsilon_0}\right)$$

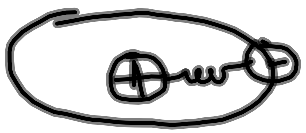


$\vdots$

ω_2

ω_1

$\omega_0 \longrightarrow 0$



Low-frequency behavior

$$\frac{\epsilon(\omega)}{\epsilon_0} = \underbrace{1 + \frac{Ne^2}{\epsilon_0 m}\sum_{j=1}^{\infty}\frac{f_j}{\omega_j^2-\omega^2}}_{\frac{\epsilon_{rest}}{\epsilon_0}} + \underbrace{\frac{Ne^2}{\epsilon_0 m}\,\frac{f_0}{(\cancel{\omega_0^2}-\omega^2-i\omega\gamma_0)}}_{\frac{Ne^2 f_0}{\epsilon_0 m \frac{\omega}{i}(\gamma_0 - i\omega)}}$$

"ω small"

$$\boxed{\epsilon(\omega) = \epsilon_{rest} + i\,\frac{Ne^2 f_0}{m\,\omega\,(\gamma_0 - i\omega)}}$$

$$\approx \frac{i}{\omega}\cdot\frac{Ne^2 f_0}{m\gamma_0} \rightarrow \sigma$$

$$\nabla\times\vec H = \underbrace{\vec J}_{\sigma\vec E} + \epsilon_{rest}\frac{\partial\vec E}{\partial t} = \left[\sigma + \epsilon_{rest}(-i\omega)\right]\vec E$$
$$= -i\omega\left[\epsilon_{rest} + \frac{i\sigma}{\omega}\right]\vec E$$

macro

$$\vec E = \vec E_0 e^{-i\omega t}$$
$$\frac{\partial\vec E}{\partial t} = (-i\omega)\,\vec E$$

Before $\epsilon = \epsilon_{rest} + \frac{i\sigma'}{\omega}$

$$\sigma' = \frac{Ne^2 f_0}{m\gamma_0}$$

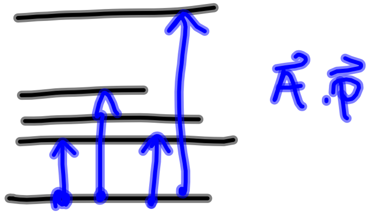
micro

Ch. 4

$\epsilon\to\infty$ "metal", $\vec E = 0$

$$\epsilon \approx \frac{i\sigma'}{\omega} \xrightarrow[\omega\to 0]{} \infty$$

$e^{-\alpha x}$



Ch. 9

Detour Sec. 6.4 Green functions of the wave equation

Max. Eqs. in Lorentz gauge

$$\nabla^2\vec{A} - \frac{1}{c^2}\frac{\partial^2\vec{A}}{\partial t^2} = -\mu_0\vec{J}, \quad \nabla^2\Phi - \frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2} = -\frac{\rho}{\epsilon_0} \Leftarrow$$

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\psi(\vec{x},t) = -4\pi f(\vec{x},t)$$

$$\psi(\vec{x},t) = \iint d^3x'\,dt'\, G(\vec{x},t;\vec{x}',t')\, f(\vec{x}',t')$$

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\psi(\vec{x},t) = \iint d^3x'\,dt' \left[\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right) G(\vec{x},t;\vec{x}',t')\right] f(\vec{x}',t') = -4\pi f(\vec{x},t)$$

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right) G(\vec{x},t;\vec{x}',t') \longrightarrow -4\pi\,\delta(\vec{x}-\vec{x}')\,\delta(t-t')$$

$$G(\vec{x}-\vec{x}', t-t') = \frac{1}{\underbrace{|\vec{x}-\vec{x}'|}_{R}} \cdot \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{i\underbrace{k}_{\frac{\omega}{c}}\underbrace{|\vec{x}-\vec{x}'|}_{R}} e^{-i\omega(t-t')} \, d\omega$$

$$\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}$$

$$= \frac{1}{|\vec{x}-\vec{x}'|} \delta\left(t' - \left(t - \frac{|\vec{x}-\vec{x}'|}{c}\right)\right)$$

<u>causality</u>

$$\nabla^2\left(\frac{1}{|\vec{x}-\vec{x}'|}\right) = -4\pi\delta(\vec{x}-\vec{x}')$$

$$\psi(\vec{x},t) = \iint d^3x'\,dt'\,\frac{1}{|\vec{x}-\vec{x}'|}\,\delta\left(t' - \left(t - \frac{|\vec{x}-\vec{x}'|}{c}\right)\right) f(\vec{x}',t')$$

$$= \int d^3x'\,\frac{1}{|\vec{x}-\vec{x}'|}\,f(\vec{x}',t')_{ret} \qquad t' = t - \frac{|\vec{x}-\vec{x}'|}{c}$$

$$\boxed{\vec{A}(\vec{x},t) = \frac{\mu_0}{4\pi}\int d^3x'\,\frac{1}{|\vec{x}-\vec{x}'|}\left[\vec{J}(\vec{x}',t')\right]_{ret}}$$

$$\boxed{= \frac{\mu_0}{4\pi}\int d^3x' \int dt'\,\frac{\vec{J}(\vec{x}',t')}{|\vec{x}-\vec{x}'|}\,\delta\left(t' + \frac{|\vec{x}-\vec{x}'|}{c} - t\right)}$$

$$\vec{J}(\vec{x}', t') = \vec{J}(\vec{x}')\, e^{-i\omega t'}$$

$$t - \frac{|\vec{x}-\vec{x}'|}{c}$$

$$\boxed{\vec{A}(\vec{x}, t) = \frac{\mu_0}{4\pi}\int d^3x' \,\frac{\vec{J}(\vec{x}')}{|\vec{x}-\vec{x}'|}\, e^{-i\omega t}\, e^{+i\frac{\omega}{c}|\vec{x}-\vec{x}'|}}$$

$$\vec{B} = \nabla\times\vec{A}, \quad \vec{E} = \frac{i}{\epsilon_0\omega}\nabla\times\vec{H}$$

always

valid if $\vec{J}(x', t') = \vec{J}(x')\, e^{-i\omega t'}$