Summary Monday's lecture

If $\rho(\vec{x},t) = \rho(\vec{x})e^{-i\omega t}$, $\vec{J}(\vec{x},t) = \vec{J}(\vec{x})e^{-i\omega t}$

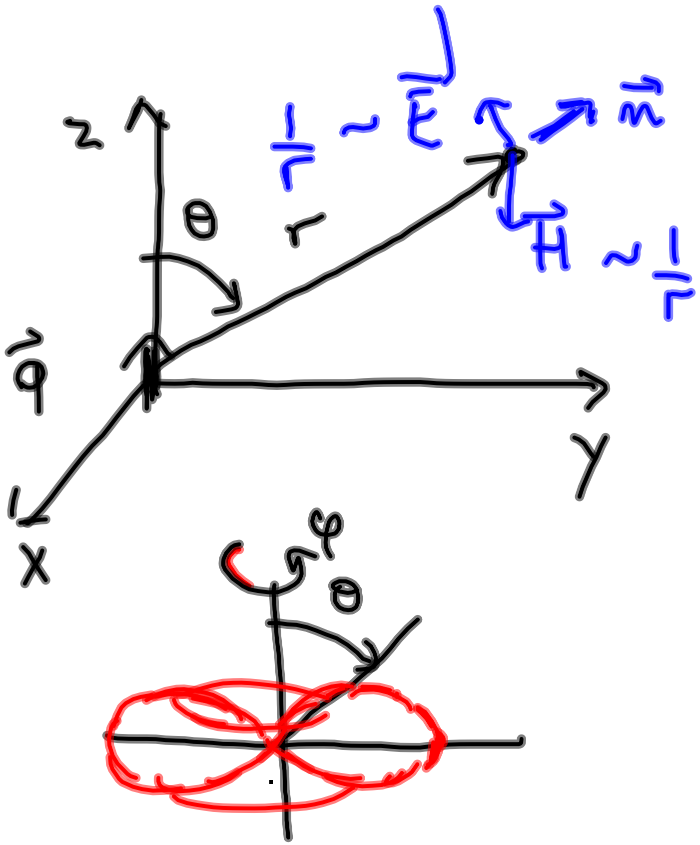
$$\hookrightarrow \vec{A}(\vec{x},t) = \vec{A}(\vec{x})e^{-i\omega t}$$

$$\vec{A}(\vec{x}) = \frac{\mu_0}{4\pi}\int d^3x' \frac{\vec{J}(\vec{x}')}{|\vec{x}-\vec{x}'|} e^{ik|\vec{x}-\vec{x}'|}$$

If $d \ll \lambda \ll r$, $\vec{A}(\vec{x}) = -i\frac{\mu_0\omega}{4\pi}\frac{e^{ikr}}{r}\vec{p}$

source $\qquad \lambda \sim \frac{1}{k}$

$$\nabla\cdot\vec{J} + \frac{\partial\rho}{\partial t} = 0 \qquad \rho(\vec{x})e^{-i\omega t}$$



$$\frac{dP}{d\Omega} = \frac{1}{2} \mathrm{Re}\left[r^2 \vec{n} \cdot (\vec{E} \times \vec{H}^*) \right]$$

general

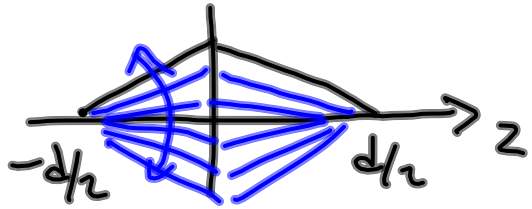
$$\frac{dP}{d\Omega} = \frac{c^2 k^4}{32\pi^2} \sqrt{\frac{\mu_0}{\epsilon_0}} |\vec{p}|^2 \sin^2\theta$$

Example:

$$I(\vec{x}) \xrightarrow[\nabla\cdot\vec{J}+\frac{\partial\rho}{\partial t}=0]{} \rho(\vec{x}) \longrightarrow \vec{p} = \int \vec{x}'\rho(\vec{x}')\,d^3x'$$

$$\nabla\cdot\vec{J} - i\omega\rho = 0$$

$$I(z) = I_0\left(1 - \frac{|z|}{d/2}\right)$$



$$\nabla\cdot\vec{I}$$

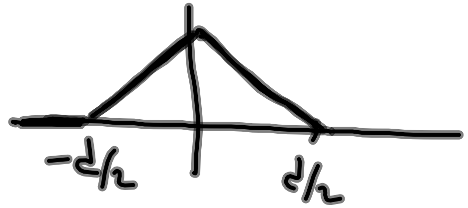
$$\vec{I} = (0,0,I(z))$$

$$\rho = \frac{\nabla\cdot\vec{I}}{i\omega} = \frac{1}{i\omega}\frac{d}{dz}\left[I_0\left(1 - \frac{|z|}{d/2}\right)\right], \quad \boxed{\rho(z) = \pm\frac{2iI_0}{\omega d}\begin{cases} + & z>0 \\ - & z<0\end{cases}}$$

$$\rho^{real}(z,t) = \mathrm{Re}\left[\rho(z)\,e^{-i\omega t}\right]$$

$\sim i$ $\quad \cos\omega t - i\sin\omega t$

$$p_z = \int_{-d/2}^{d/2} z\,\rho(z)\,dz = \frac{i I_0 d}{2\omega}$$



$$\frac{dP}{d\Omega} = \frac{I_0^2}{128\pi^2}\sqrt{\frac{\mu_0}{\epsilon_0}}(kd)^2 \sin^2\theta$$

$$\int d\omega\, f(\omega)$$

ω_1 ω_2

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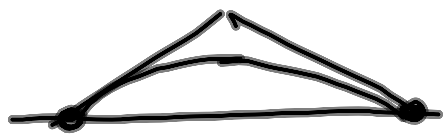
HW

9.4 linear antenna

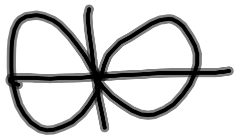
$$\vec{A}(\vec{x}) = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{x}')\, e^{ik|\vec{x}-\vec{x}'|}}{|\vec{x}-\vec{x}'|} d^3x'$$

$$\sin\left(\frac{kd}{2} - k|z|\right), \quad |z| < d/2$$

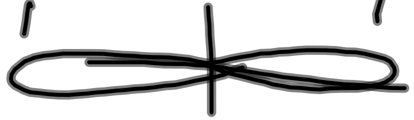


$$\frac{dP}{d\Omega} \propto \left| \frac{\cos\left[\frac{kd}{2}\cos\theta\right] - \cos\left(\frac{kd}{2}\right)}{\sin\theta} \right|^2 \neq \sin^2\theta$$

$kd = \pi$



, $kd = 2\pi$



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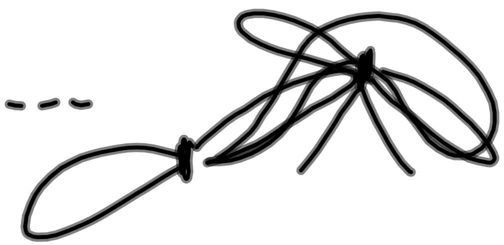
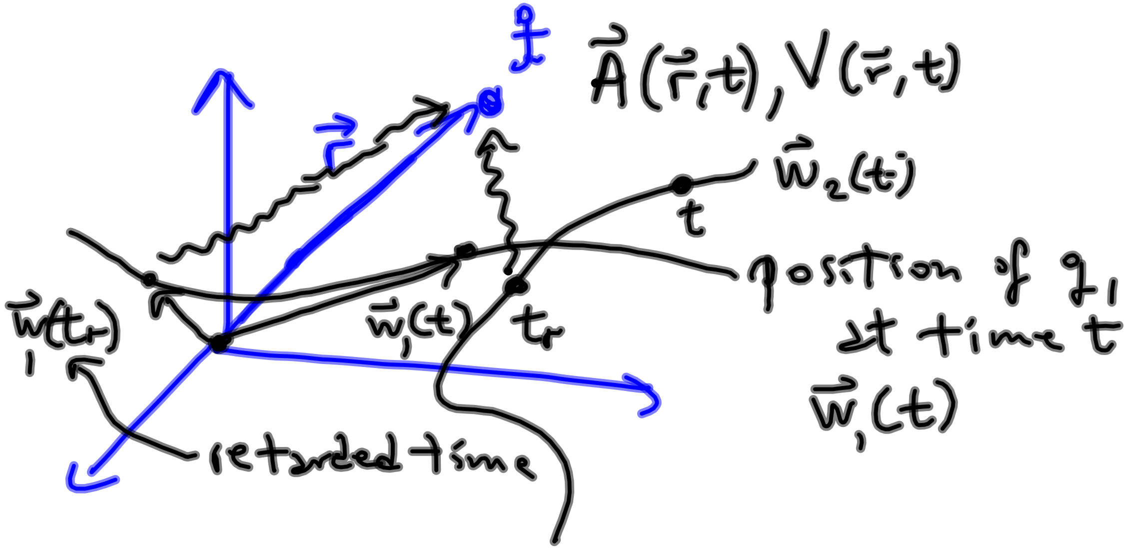


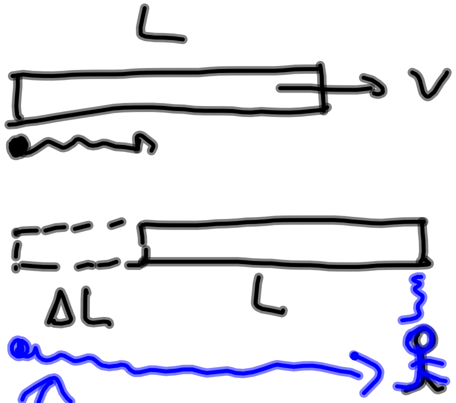
10.3 third edition

9.2 second edition

Lienard - Wiechert Potentials

Radiation from a Point Charge





$$L' = L + \Delta L$$

$$\left.\frac{L'}{c} = \frac{L' - L}{v}\right\} \rightarrow \boxed{L' = \frac{L}{1 - \frac{v}{c}}}$$

$$V(\vec{r},t) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{r}',t_r)\, d^3r'}{|\vec{r} - \vec{w}(t_r)|} \sim \frac{1}{4\pi\epsilon_0}\frac{1}{|\vec{\mathscr{r}}|} \underbrace{\overbrace{\int \rho(r',t_r)\, d^3r'}^{\neq Q}}_{\dfrac{Q}{\left(1 - \frac{\hat{\mathscr{r}}\cdot\vec{v}}{c}\right)}}$$

$$\boxed{V(\vec{r},t) = \frac{1}{4\pi\epsilon_0}\frac{q}{|\vec{\mathscr{r}}|\left(1-\frac{\hat{\mathscr{r}}\cdot\vec{v}}{c}\right)}}$$

$$\vec{v} \to 0$$

$$\vec{A}(\vec{r},t)$$