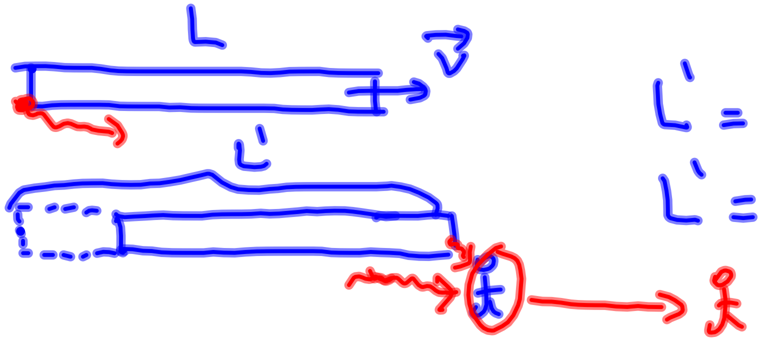
$$V(\vec{r},t) = \frac{1}{4\pi\epsilon_0 |\vec{\mathscr{r}}|}\underbrace{\int \rho(\vec{r}', t_r)\, d^3r'}_{\neq Q}$$

$\vec{\mathscr{r}} = \vec{r} - \vec{w}(t_r)$

$\hat{\mathscr{r}}\,,\ \hat{r}$

$|\vec{\mathscr{r}}| = \mathscr{r}\ ,\ r = |\vec{r}|$



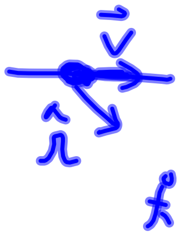
$$\left.\begin{aligned} L' &= L + vt \\ L' &= ct \end{aligned}\right\} \quad \underbrace{\frac{L'}{c} = \frac{L'-L}{v}}\ = t$$

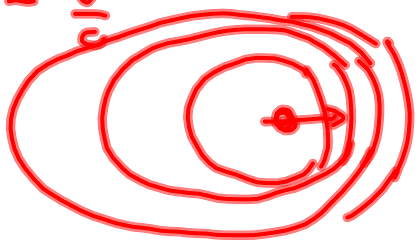
$$\boxed{L' = \frac{L}{1 - \frac{v}{c}}}$$

indep. of L

indep. of the actual position

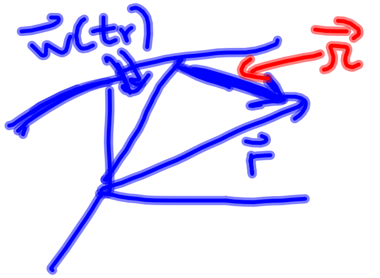
$$Q_{app} = L'\frac{Q}{L} = \frac{Q}{1 - \frac{v}{c}}$$





$$\frac{1}{1 - \frac{\hat{\mathscr{r}}\cdot\vec{v}}{c}}$$

$$V(\vec{r},t) = \frac{q}{4\pi\epsilon_0 |\vec{\mathscr{r}}|} \frac{1}{\left(1 - \frac{\hat{\mathscr{r}}\cdot\vec{v}}{c}\right)} \underset{|\vec{v}|\to 0}{\rightsquigarrow} \frac{q}{4\pi\epsilon_0 r}$$



$\rho\vec{v}$

$$\vec{A}(\vec{r},t) = \frac{\vec{v}(t_r)}{c^2} V(\vec{r},t) \underset{|\vec{v}|\to 0}{\rightsquigarrow} 0$$

$$\vec{E} = -\nabla V - \frac{\partial \vec{A}}{\partial t} \quad ; \quad \vec{B} = \nabla \times \vec{A}$$

Example 10.3

$$\vec{w} = \vec{v}\, t \quad \leftarrow \text{constant}$$

$$\mathscr{r} = |\vec{\mathscr{r}}| = |\vec{r} - \vec{w}| \quad \leftarrow \vec{w}(t_r)$$

$$|\vec{r} - \vec{w}| = |\vec{r} - \vec{v}\, t_r| = c\,(t - t_r)$$

$$|\vec{r} - \vec{v}\, t_r|^2 = c^2 (t - t_r)^2$$

$$r^2 - 2\vec{r}\cdot\vec{v} + v^2 t_r^2 = c^2 (t^2 - 2 t t_r + t_r^2)$$

$$t_r = \frac{--- \pm \sqrt{---}}{---}$$

$$\nabla t_r \neq 0 \qquad \nabla t = 0$$

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$$\boxed{\nabla t_r = \frac{-\vec{\mathscr{r}}}{\mathscr{r} c - \vec{\mathscr{r}}\cdot\vec{v}}}$$

$$\nabla V = \nabla\left[\frac{qc}{4\pi\epsilon_0(\mathfrak{r}c - \vec{\mathfrak{r}}\cdot\vec{v})}\right] = \frac{qc}{4\pi\epsilon_0}\left[-\frac{1}{(\mathfrak{r}c-\vec{\mathfrak{r}}\cdot\vec{v})^2}\right]\nabla(\mathfrak{r}c-\vec{\mathfrak{r}}\cdot\vec{v})$$

$$\nabla(\mathfrak{r}c) = c\nabla\mathfrak{r} = c\nabla[c(t-t_r)] = -c^2\nabla t_r \neq 0$$

$\nabla t = 0$, $\nabla t_r \neq 0$

$$\nabla(\vec{\mathfrak{r}}\cdot\vec{v}) \qquad \frac{\partial}{\partial x}\vec{v}(t_r) = \frac{\partial\vec{v}}{\partial t_r}\frac{\partial t_r}{\partial x}$$

$\frac{\partial\vec{v}}{\partial t_r} \to \vec{a}(t_r)$

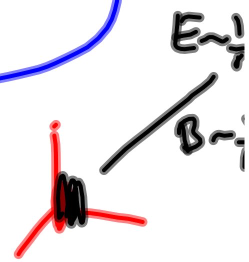
$$\frac{\partial}{\partial t}\vec{A}(\vec{r},t)$$

$$\nabla\times\vec{A}(\vec{r},t)$$

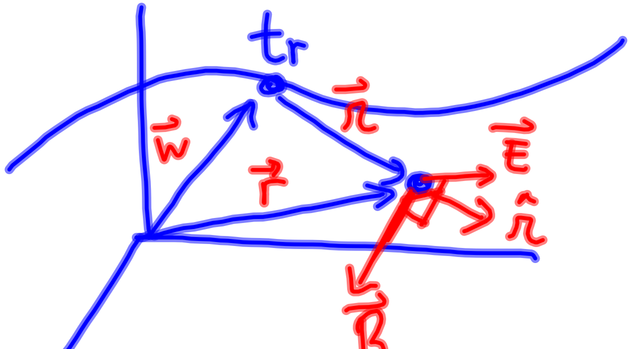
$$\vec{E}(\vec{r},t) = \frac{q}{4\pi\epsilon_0}\frac{\mathfrak{r}}{(\vec{\mathfrak{r}}\cdot\vec{u})^3}\left[\vec{u}(c^2-v^2)+\vec{\mathfrak{r}}\times(\vec{u}\times\vec{a})\right]$$

$$\vec{u} = c\hat{\mathfrak{r}} - \vec{v}$$

$\frac{1}{r^2}$ $\qquad$ $\frac{1}{r}$



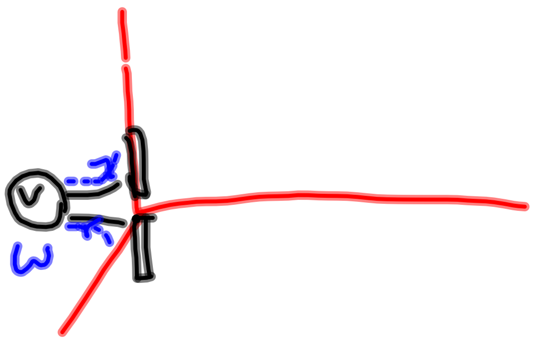
$$\vec{B} = \nabla \times \vec{A} = \frac{1}{c}(\hat{\imath} \times \vec{E})$$



for any $\vec{\imath}$

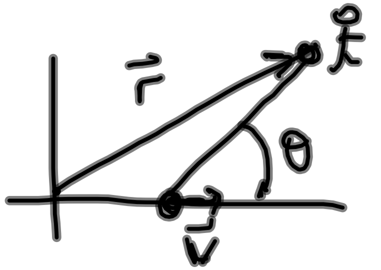
large distances

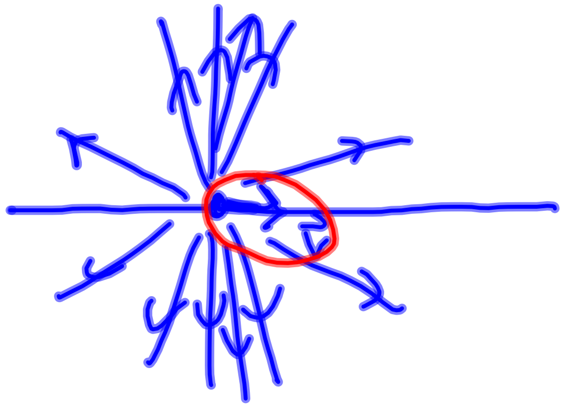
plane waves

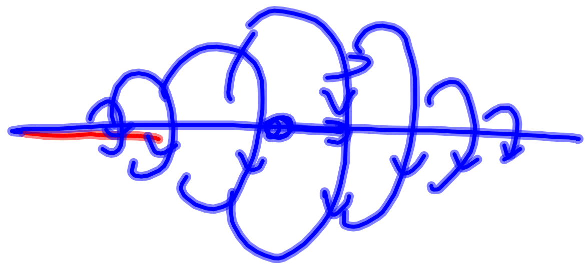


Example

$$\left.\begin{array}{l}\vec{a}=0\\ \vec{w}=\vec{v}t\end{array}\right| \longrightarrow \vec{E}=\frac{q}{4\pi\epsilon_0}\cdot\frac{\vec{r}-\vec{v}t}{|\vec{r}-\vec{v}t|^3}\cdot\frac{1-\frac{v^2}{c^2}}{\left(1-\frac{v^2}{c^2}\sin^2\theta\right)^{3/2}}$$





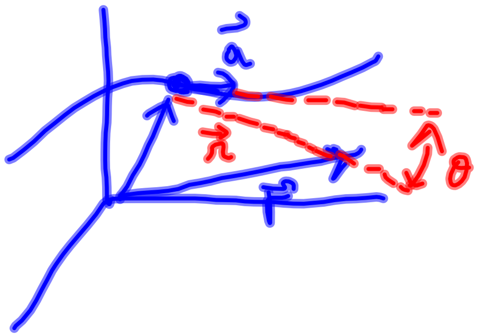


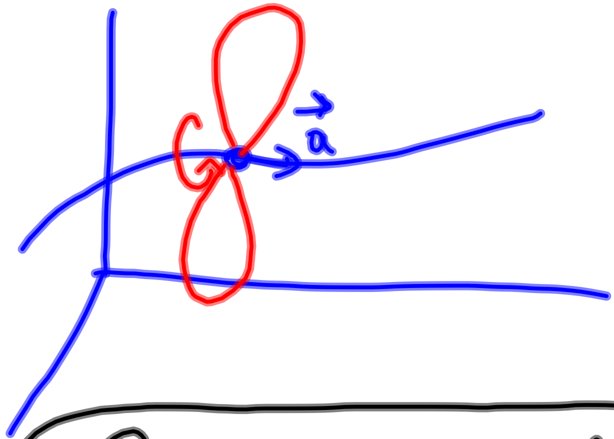
Power radiated by a point charge

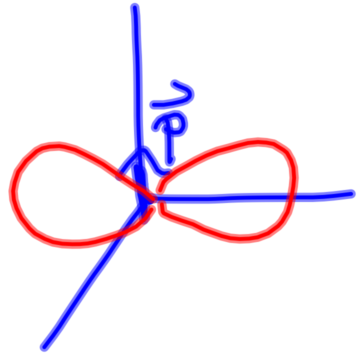
$\vec{E}\ \vec{B}$

$$\vec{S} = \frac{1}{\mu_0 c}\underbrace{|\vec{E}|^2}_{|\vec{E}|\cdot\frac{|\vec{E}|}{c}}\hat{\imath}$$

Example: $\vec{v}=0,\ \vec{a}\neq 0$; $\vec{S} = \frac{1}{4\pi\epsilon_0}\cdot\frac{q^2}{4\pi c^3}\cdot\frac{a^2\sin\theta}{\imath^2}\hat{\imath}$







$$P = \frac{1}{4\pi\epsilon_0} \cdot \frac{2}{3} \cdot \frac{q^2 a^2}{c^3}$$ Larmor formula