Ch. 11 Theory of Relativity

Galilean transformations

K, K' systems of coordinates, $\vec{v}$

$$\begin{cases} \vec{x}' = \vec{x} - \vec{v}t \\ t' = t \end{cases}$$

At $t=0$, $K = K'$

K' $$m_i \frac{d\vec{v}_i'}{dt'} = -\nabla_i' \sum_j V_{ij}(|\vec{x}_i' - \vec{x}_j'|)$$

$m\vec{a} = \vec{F} = -\nabla\phi$

K $$m_i \frac{d\vec{v}_i}{dt} = -\nabla_i \sum_j V_{ij}(|\vec{x}_i - \vec{x}_j|)$$

$$\nabla^2 \Phi - \frac{1}{c^2}\frac{\partial^2 \Phi}{\partial t^2} = -\frac{\rho}{\epsilon_0} \qquad \nabla\cdot\vec{A} + \frac{1}{c^2}\frac{\partial \Phi}{\partial t} = 0$$

$$\nabla^2 \vec{A} - \frac{1}{c^2}\frac{\partial^2 \vec{A}}{\partial t^2} = -\mu_0 \vec{J}$$

$$\left[\sum_{i=1}^{3}\frac{\partial^2}{\partial x_i^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right]\psi = 0$$

galilean Transf.

$$\left[\sum_{i=1}^{3}\frac{\partial^2}{\partial x_i^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{2}{c^2}(\vec{v}\cdot\vec{\nabla})\frac{\partial}{\partial t} - \frac{1}{c^2}(\vec{v}\cdot\vec{\nabla})(\vec{v}\cdot\vec{\nabla})\right]\psi = 0$$

air, water, ... <u>medium</u> ether

* Max. Eq. are wrong. However, Max. Eq. are very accurate

* Max. Eqs. are right, but there is an ether.
  no evidence of ether

* Max. Eqs. are right, there is no ether
  ↳ Newton Eqs. are incomplete
  "generalized galilean transf" needed

$c = c'$

$$\left(\sum_{i=1}^{3} \frac{\partial^2}{\partial x_i^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\psi = 0$$

1D

$$\left.\begin{aligned} x' &= x - vt \\ t' &= t \end{aligned}\right\} \rightarrow \begin{aligned} x &= x' + vt' \\ t &= t' \end{aligned} \leftarrow$$

$$\frac{\partial}{\partial t'}\psi = \underbrace{\frac{\partial x}{\partial t'}}_{v}\frac{\partial \psi}{\partial x} + \underbrace{\frac{\partial t}{\partial t'}}_{1}\frac{\partial \psi}{\partial t} = v\frac{\partial \psi}{\partial x} + \frac{\partial \psi}{\partial t}$$

$$\frac{\partial}{\partial x'} \rightarrow \frac{\partial}{\partial x}$$

$$\frac{\partial}{\partial t'} \longrightarrow v\frac{\partial}{\partial x} + \frac{\partial}{\partial t} \qquad \text{1D}$$

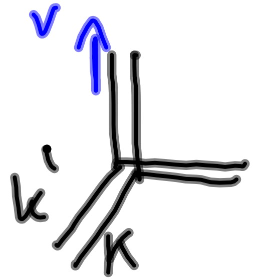
$$\frac{\partial}{\partial t'} \longrightarrow \vec{v}\cdot\nabla + \frac{\partial}{\partial t} \qquad \text{3D}$$

$$\frac{\partial^2}{\partial x'^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t'^2} \longrightarrow \frac{\partial^2}{\partial x^2} - \frac{1}{c^2}\left(\frac{\partial}{\partial t} + v\frac{\partial}{\partial x}\right)\left(\frac{\partial}{\partial t} + v\frac{\partial}{\partial x}\right)$$

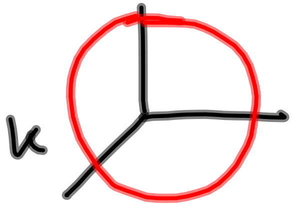
$$= \frac{\partial^2}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} \underbrace{- \frac{2v}{c^2}\frac{\partial}{\partial x}\frac{\partial}{\partial t} - \frac{v^2}{c^2}\frac{\partial^2}{\partial x^2}}_{\text{unwanted}}$$

$c = c'$

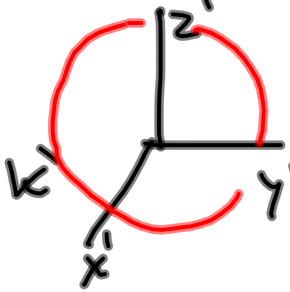
11.3 Lorentz transformations



At $t=0$, a pulse of light is switched on and off



$$x^2 + y^2 + z^2 = R^2 = (ct)^2$$



$$(x')^2 + (y')^2 + (z')^2 = (ct')^2$$

$$\boxed{c^2t^2 - (x^2+y^2+z^2) = c^2t'^2 - (x'^2+y'^2+z'^2)}$$

$$\left\{\begin{array}{l} ct' = x_0' = \gamma(\overbrace{x_0}^{ct} - \beta z) \\ z' = \gamma(z - \beta x_0) \\ x' = x \\ y' = y \end{array}\right. \quad , \gamma = \frac{1}{\sqrt{1-\beta^2}} \quad , \beta = \frac{v}{c}$$

$$c^2t'^2 = \gamma^2(x_0 - \beta z)^2$$

$$z'^2 = \gamma^2(z - \beta x_0)^2$$

$$c^2t'^2 - z'^2 = c^2t^2 - z^2$$

$$\frac{\partial\psi}{\partial t'} = \frac{\partial x}{\partial t'}\frac{\partial\psi}{\partial x} + \frac{\partial t}{\partial t'}\frac{\partial\psi}{\partial t}$$

$$x_0' = \gamma(x_0 - \beta x)$$
$$x' = \gamma(x - \beta x_0)$$

new T̂ that I am testing

$$x_0 = \gamma(x_0' + \beta x_1')$$
$$x = \gamma(x' + \beta x_0')$$

$$\frac{\partial x}{\partial x_0'} = \frac{1}{c}\frac{\partial x}{\partial t'} = \gamma\beta \;,\; \frac{\partial x_0}{\partial x_0'} = \gamma = \frac{\partial t}{\partial t'}$$

Max. Eqs. are invariant under the Lorentz transformations.

4-vectors

$$A'_0 = \gamma(A_0 - \beta A_1)$$

if $$A'_1 = \gamma(A_1 - \beta A_0)$$

$$A'_2 = A_2$$

$$A'_3 = A_3$$

$\rightarrow (A^0, \vec{A})$ is a 4-vector

$$A_0'^2 - (A_1'^2 + A_2'^2 + A_3'^2) = A_0^2 - (A_1^2 + A_2^2 + A_3^2)$$

norm

scalar product $(A,B) = A'_0 B'_0 - \vec{A}' \cdot \vec{B}' = A_0 B_0 - \vec{A} \cdot \vec{B}$

$$x'^{\alpha} = x'^{\alpha}(x_0, x_1, x_2, x_3) \quad \alpha = 0, 1, 2, 3$$

$$E'_{\alpha} = \sum_{\beta=0}^{3} \frac{\partial x'^{\alpha}}{\partial x^{\beta}} E_{\beta} \;, \quad E'_{\alpha} = \sum_{\beta} \frac{\partial x^{\beta}}{\partial x'^{\alpha}} E_{\beta}$$

covariant

$$B'_{\alpha} = \frac{\partial x^{\beta}}{\partial x'^{\alpha}} B_{\beta}$$

contravariant

$$A'^{\alpha} = \frac{\partial x'^{\alpha}}{\partial x^{\beta}} A^{\beta}$$

contravariant tensor of rank two

$$F'^{\alpha\beta} = \frac{\partial x'^{\alpha}}{\partial x^{\gamma}} \frac{\partial x'^{\beta}}{\partial x^{\delta}} F^{\gamma\delta}$$