Last Eq. of previous lecture (12.63)

Scalar product $\sum_\alpha B_\alpha A^\alpha$ ← contravariant

$\vec{a}\cdot\vec{b}$

cov.

invariant under a Lorentz transf.

$$A^\alpha = (A^0, \vec{A}) \longrightarrow A_\alpha = (A^0, -\vec{A})$$

$$g_{\mu\nu} = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$$

$$\partial^\alpha = \left(\frac{\partial}{\partial x^0}, -\vec{\nabla}\right), \quad \partial_\alpha = \left(\frac{\partial}{\partial x^0}, +\vec{\nabla}\right)$$

$$\partial_\alpha A^\alpha = \frac{\partial A^0}{\partial x^0} + \vec{\nabla}\cdot\vec{A} = \frac{1}{c}\frac{\partial \Phi}{\partial t} + \vec{\nabla}\cdot\vec{A} = 0$$

$x^0 = ct$

$$\boxed{\partial_\alpha A^\alpha = 0}$$

$A^\alpha = (A^0, \vec{A})$

A^0: Φ

$\vec{A}$: vector field

$$\Box \underset{\text{def}}{\equiv} \partial_\alpha \partial^\alpha = \frac{\partial^2}{\partial x^{0\,2}} - \nabla^2 = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2$$

$$\frac{1}{c^2}\frac{\partial^2 \vec{A}}{\partial t^2} - \nabla^2 \vec{A} = \frac{4\pi}{c}\vec{J} \rightarrow \Box \vec{A} = \frac{4\pi}{c}\vec{J}$$

$$\frac{1}{c^2}\frac{\partial^2 \Phi}{\partial t^2} - \nabla^2 \Phi = 4\pi\rho \rightarrow \Box \Phi = \frac{4\pi}{c}(\rho c)$$

$$\boxed{\Box A^\alpha = \frac{4\pi}{c} J^\alpha}$$

$$\boxed{\partial_\alpha A^\alpha = 0}$$

manifestly covariant

$$\vec{E} = -\frac{1}{c}\frac{\partial \vec{A}}{\partial t} - \nabla\Phi$$

$$\vec{B} = \nabla \times \vec{A}$$

$$E_x = -\frac{1}{c}\frac{\partial A_x}{\partial t} - \frac{\partial \Phi}{\partial x} = -\left(\partial^0 A^1 - \partial^1 A^0\right)$$

$$B_x = \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} = -\left(\partial^2 A^3 - \partial^3 A^2\right) \qquad \partial^\alpha = \left(\frac{\partial}{\partial x^0}, -\vec{\nabla}\right)$$

antisymmetric field strength tensor

$$F^{\alpha\beta} = \partial^\alpha A^\beta - \partial^\beta A^\alpha$$

$$F^{\alpha\beta} = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{pmatrix}$$

$$F^{\alpha\beta} = \begin{pmatrix} 0 & -B_x & -B_y & -B_z \\ B_x & 0 & E_z & -E_y \\ B_y & -E_z & 0 & E_x \\ B_z & E_y & -E_x & 0 \end{pmatrix}$$

$$\nabla \cdot \vec{E} = 4\pi\rho$$

$$\nabla \times \vec{B} - \frac{1}{c}\frac{\partial \vec{E}}{\partial t} = \frac{4\pi}{c}\vec{J}$$

$$\boxed{\partial_\alpha F^{\alpha\beta} = \frac{4\pi}{c} J^\beta}$$

$\beta = 0$

$$\partial_\alpha F^{\alpha 0} = \frac{4\pi}{c} J^0$$

$$\cancel{\partial_0 F^{00}} + \underbrace{\partial_1}_{\frac{\partial}{\partial x}} \underbrace{F^{10}}_{E_x} + \underbrace{\partial_2}_{\frac{\partial}{\partial y}} \underbrace{F^{20}}_{E_y} + \underbrace{\partial_3}_{\frac{\partial}{\partial z}} \underbrace{F^{30}}_{E_z} = \frac{4\pi}{\cancel{c}} J^0$$

$$J^0 = \cancel{c}\rho$$

$$\equiv \nabla \cdot \vec{E} = 4\pi\rho$$

$$\nabla \cdot \vec{B} = 0 \ , \quad \nabla \times \vec{E} + \frac{1}{c}\frac{\partial \vec{B}}{\partial t} = 0$$

$$\boxed{\partial_\alpha \mathcal{F}^{\alpha\beta} = 0} \qquad \boxed{\partial_\alpha F^{\alpha\beta} = \frac{4\pi}{c} J^\beta}$$

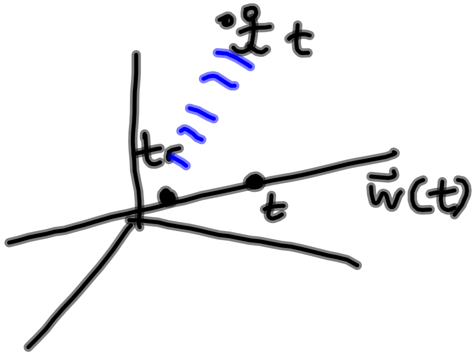
$$F'^{\alpha\beta} = \sum_{\gamma\delta} \frac{\partial x'^\alpha}{\partial x^\gamma} \frac{\partial x'^\beta}{\partial x^\delta} F^{\gamma\delta}$$

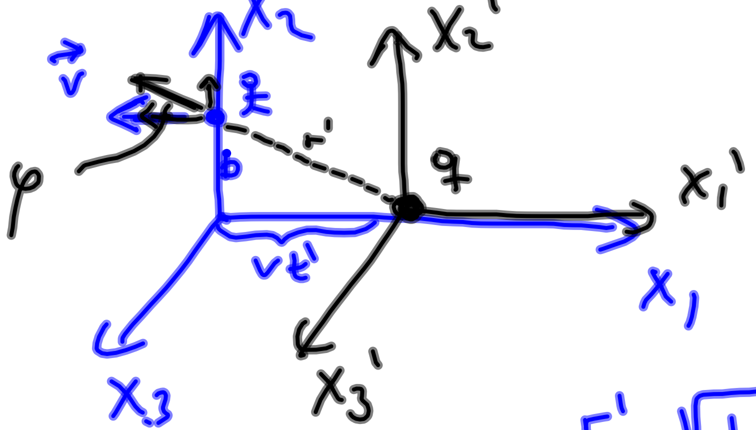
$$\left[\begin{array}{l} x'^0 = \gamma(x^0 - \beta x^1) \\ x'^1 = \gamma(x^1 - \beta x^0) \\ x'^2 = x^2 \\ x'^3 = x^3 \end{array}\right.$$

$$\underbrace{F'^{10}}_{E'_x} = E_x \ , \quad E'_y = \gamma(E_y - \beta B_z)$$

$$\underbrace{F'^{21}}_{B'_z} = \gamma(B_z - \beta E_y)$$

$$\ldots\ldots$$





$$r' = \sqrt{b^2 + (vt')^2}$$

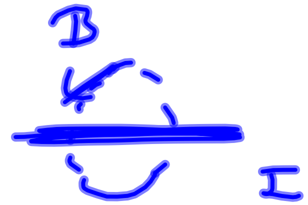
$$E_1' = -\frac{q}{r'^2} \cdot \frac{vt'}{r'} = \frac{-q\,vt'}{r'^3} \; ; \quad E_3' = 0$$

$$E_2' = \frac{qb}{r'^3}$$

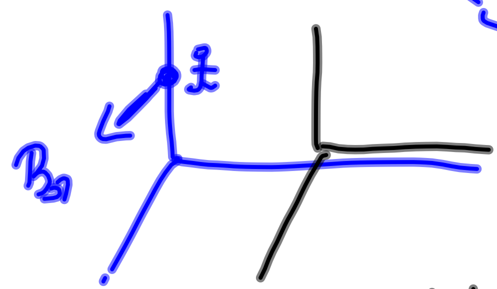
$$B_1' = B_2' = B_3' = 0$$

$$t' = \gamma\left(t - \cancel{\frac{v}{c^2} x_1'}\right)$$

$$E_1 = \frac{-q v \gamma t}{(b^2 + v^2\gamma^2 t^2)^{3/2}} \qquad E_1' = E_1$$



$$E_2 = \frac{\gamma q b}{(b^2 + v^2\gamma^2 t^2)^{3/2}}$$



$$E_3 = 0$$

$$B_1 = B_2 = 0 \,, \quad B_3 = \frac{\gamma q \beta b}{(b^2 + v^2\gamma^2 t^2)^{3/2}}$$