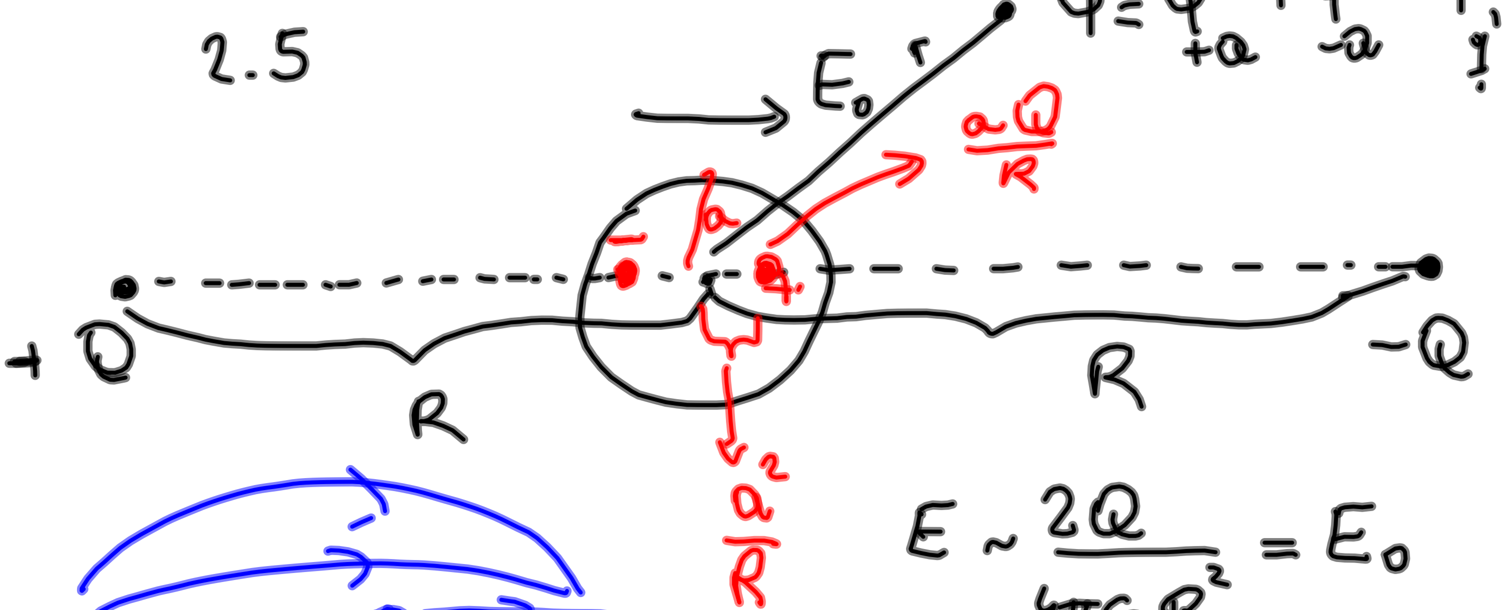
Monday $10^{45}-12$, room 307

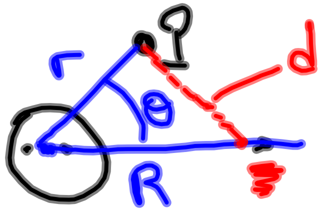
2.5

$$\phi = \phi_{+Q} + \phi_{-Q} + \phi_{q'} + \phi_{-q'}$$



$$E \sim \frac{2Q}{4\pi\epsilon_0 R^2} = E_0$$

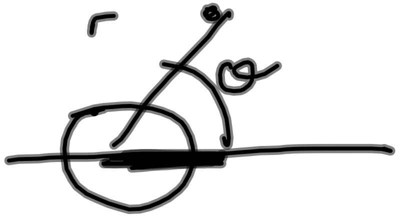
$$Q \to \infty$$
$$R \to \infty$$
$$\frac{Q}{R^2} = \frac{4\pi\epsilon_0 E_0}{2}$$



$$d = \sqrt{r^2 + R^2 - 2rR\cos\theta}$$

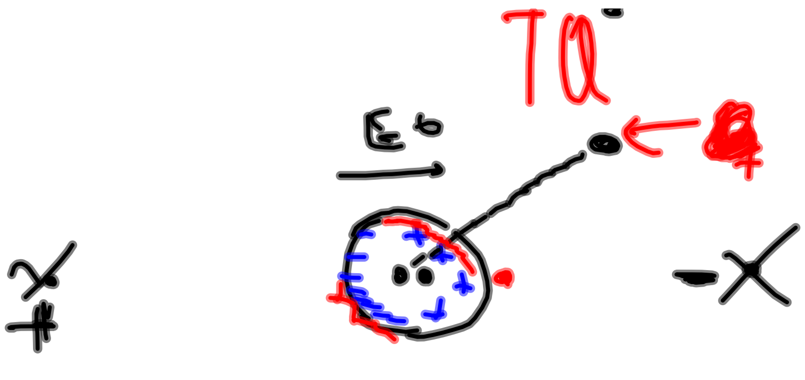
$$\Phi_{-Q} = \left(-\frac{Q}{4\pi\epsilon_0}\right)\frac{1}{d}$$

$$-\frac{Q}{4\pi\epsilon_0}\,\frac{1}{d} = \frac{1}{R}\,\underbrace{\frac{1}{\sqrt{1 + \left(\frac{r}{R}\right)^2 - \frac{2r}{R}\cos\theta}}} \approx \overbrace{\frac{1}{R}}^{-\frac{Q}{4\pi\epsilon_0}}\left(1 + \frac{1}{2}\,\frac{2r}{R}\cos\theta\right)$$



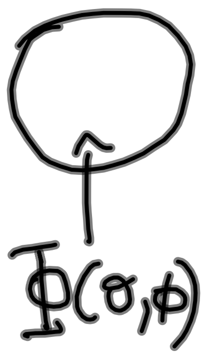
$$\frac{1}{\sqrt{1+u}} \approx 1 - \frac{1}{2}u \quad , \; u \text{ small}$$

$$\boxed{\Phi = -E_0\left(r - \frac{a^3}{r^2}\right)\cos\theta} \; , \quad E_0 = \frac{2Q}{4\pi\epsilon_0 R^2}$$



$$E = \lim_{\Delta q \to 0} \frac{\Delta F}{\Delta q}$$

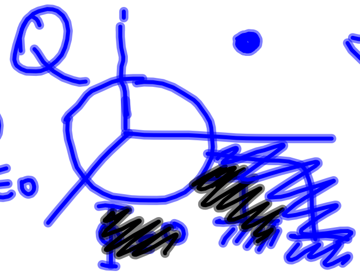
2.6 Green function of a sphere



$G_D(\vec{x}, \vec{x}')$

G_D at sphere $= 0$

$$\boxed{\begin{gathered} \nabla^2 G_D = -4\pi\,\delta(\vec{x} - \vec{x}') \\ G_D = 0 \text{ sphere} \end{gathered}}$$

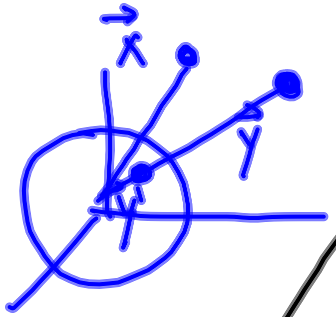


$\phi \to G_D$

$q \to 4\pi\epsilon_0$

$$\nabla^2 \Phi = -\frac{\rho}{\epsilon_0} = -\frac{q\,\delta(\vec{x} - \vec{x}')}{\epsilon_0}$$

$$G_D(\vec{x},\vec{x}') = \frac{1}{|\vec{x}-\vec{x}'|} - \frac{(a/x')}{\left|\vec{x}-\frac{a^2}{x'^2}\vec{x}'\right|}\,.$$



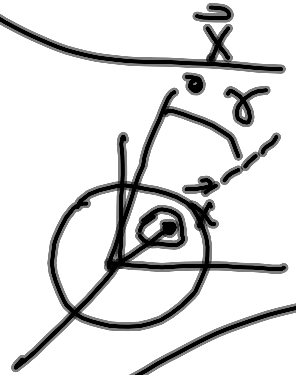
$$\phi = \int_V \rho G + \cancel{\int_S \phi \frac{\partial G}{\partial n}} + \cancel{\int_S G \frac{\partial \phi}{\partial n}}$$

$\frac{\partial G_D}{\partial n}$ at the surface

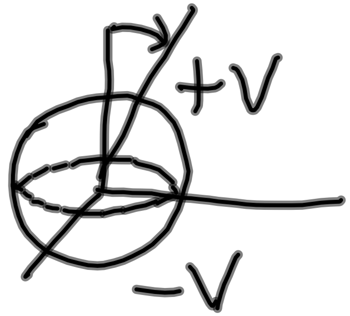
G_D

G_N

$$\Phi(\vec{x}) = \frac{1}{4\pi}\int \underbrace{d\Omega'}_{\int_0^{2\pi} d\phi' \int_0^{\pi} d\theta' \sin\theta'} a^2\, \Phi(\theta',\phi')\, \frac{(a^2-x^2)}{a\left(a^2+x^2-2ax\cos\gamma\right)^{3/2}}$$

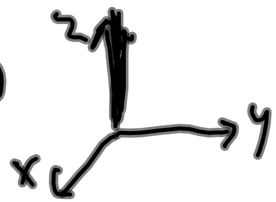


2.7



$$\int_0^{\pi} = \underbrace{\int_0^{\pi/2}}_{+V} + \underbrace{\int_{\pi/2}^{\pi}}_{-V}$$

$\theta = 0$



$$\Phi(z, \theta=0, \phi) = V\left(1 - \frac{(z^2-a^2)}{z\sqrt{z^2+a^2}}\right) \quad (2.22)$$

large z $$\approx \frac{3}{2}\frac{Va^2}{z^2}$$