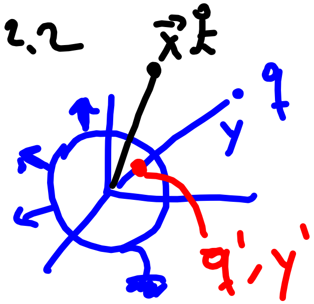
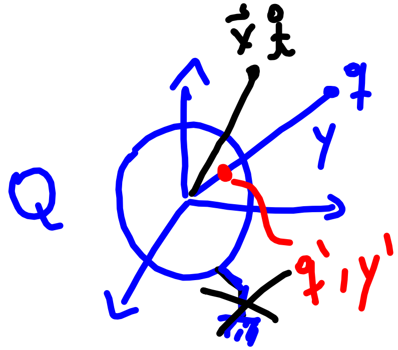
2.3 Point charge in the presence of a charged, insulated, conducting sphere.

Sec. 2.2

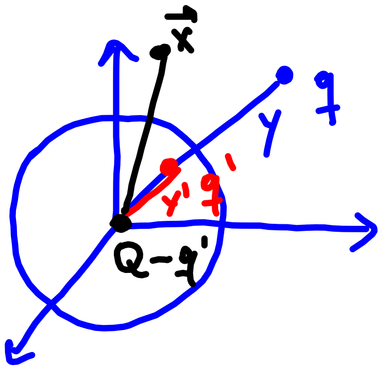


Equilibrium



$\int_S \sigma\, da = q'$

First, connect to the ground without Q.
Equilibrium.
Unconnect.
Distribute $Q - q'$ (uniform)



$$\Phi(\vec{x}) = \frac{1}{4\pi\epsilon_0}\left[\frac{q}{|\vec{x}-\vec{y}|} - \frac{\frac{a}{y}q}{\left|\vec{x}-\frac{a^2}{y^2}\vec{y}\right|} + \underbrace{\frac{Q-\left(-\frac{a}{y}q\right)}{|\vec{x}|}}_{\text{Provides the charge } Q \text{ (net) at the ball}}\right]$$

real

image

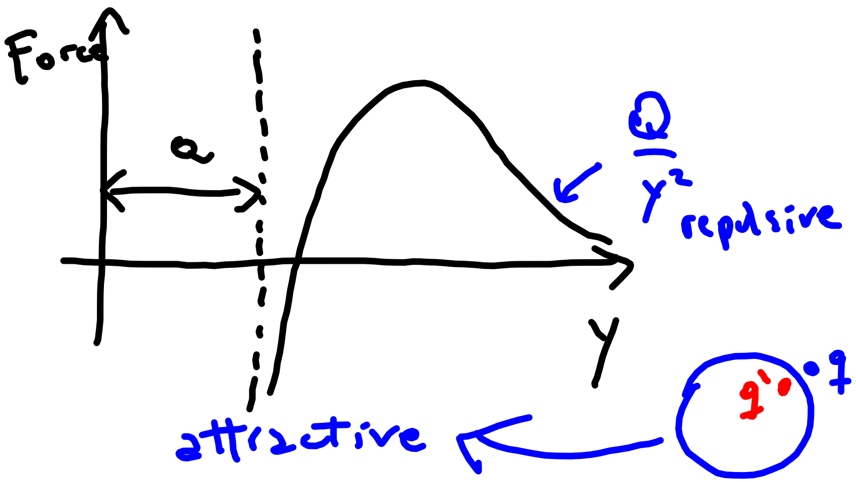
obs. point

location of q

location of q'

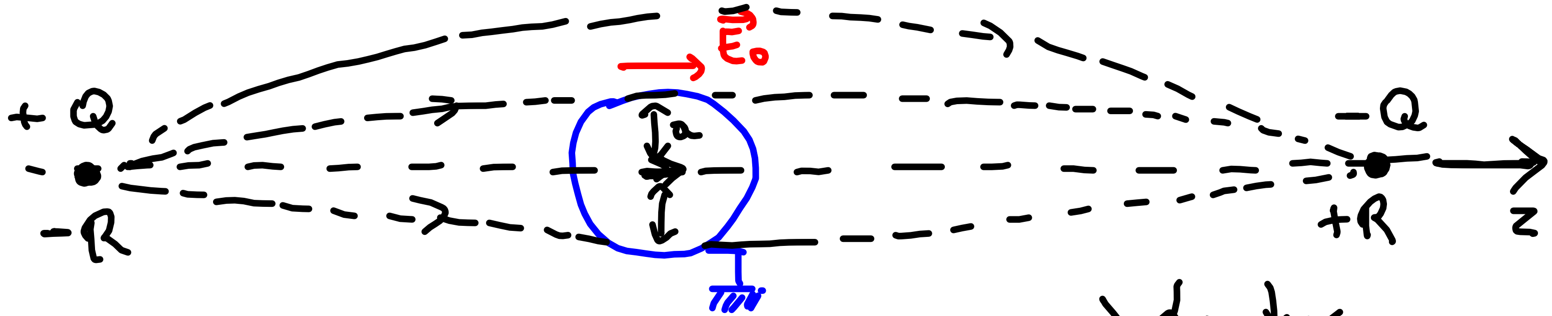
Parameters: q, y, a, Q

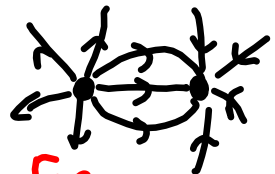
Force at q due to q' and $Q-q'$ [Eq (2.9)]



Work functions of metals

2.5 Conducting sphere in a uniform electric field

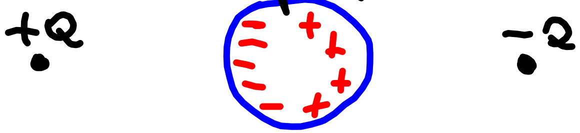


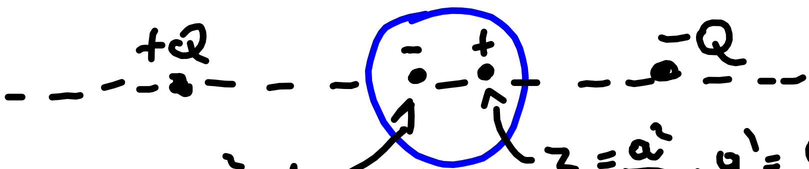


$R \to \infty \qquad \dfrac{Q}{R^2} = \text{constant}$

$Q \to \infty \qquad\quad = 2\pi \epsilon_0 E_0 \; ; \; E_{\text{sphere}} = 2\,\dfrac{Q}{4\pi\epsilon_0 R^2} = E_0$

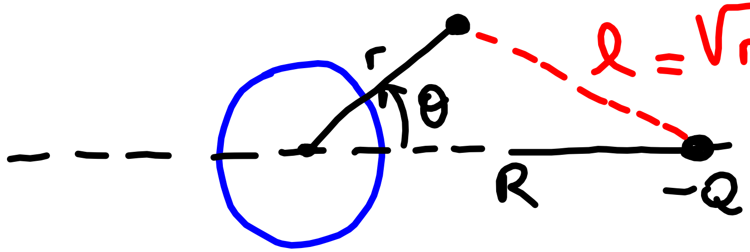
dipole





$z = -\dfrac{a^2}{R}, \; q' = -\dfrac{aQ}{R} \qquad z = \dfrac{a^2}{R}, \; q' = \dfrac{aQ}{R}$

$$\Phi(\vec{x}) = \Phi_1 + \Phi_2 + \Phi_3 + \Phi_4$$
$$Q \quad -Q \quad q'_Q \quad q'_{-Q}$$



$$\ell = \sqrt{r^2 + R^2 - 2rR\cos\theta}$$

$$\Phi_2 = -\frac{Q}{4\pi\epsilon_0}\frac{1}{\sqrt{r^2+R^2-2rR\cos\theta}} \approx -\frac{Q}{4\pi\epsilon_0}\frac{1}{R}\left[1 + \frac{r}{R}\cos\theta\right]$$

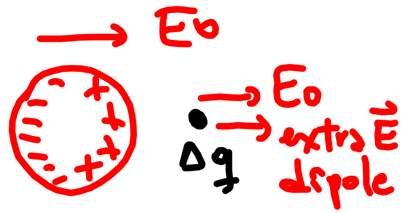
Same procedure for Φ_1, Φ_3, Φ_4

$\sim \frac{r}{R^2}$ "far" $\qquad \sim \frac{a^3}{R^2 r^2}$ "close"

$$\Phi = \underbrace{\frac{Q}{4\pi\epsilon_0 R^2}}_{E_0/2}\left[-2r\cos\theta + \frac{2a^3}{r^2}\cos\theta\right] + \cdots$$

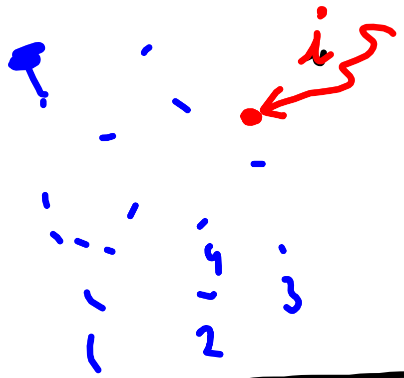
$$\boxed{\Phi = -E_0\left(\underbrace{r\cos\theta}_{z} - \frac{a^3}{r^2}\cos\theta\right)}$$

constant $\vec{E}_0$ dipolar contribution



$$E_{\text{along the z axis}} = -\frac{\partial}{\partial z}\Phi = E_0\left(1+\frac{2a^3}{z^3}\right)$$

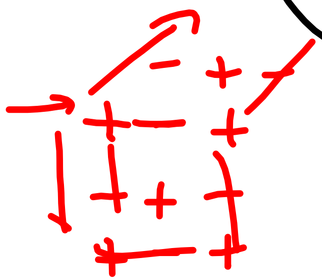
1.11 Electrostatic Potential energy



$$W_i = q_i \, \Phi(\vec{x}_i)$$

$$\hookrightarrow \frac{1}{4\pi\epsilon_0}\sum_{j=1}^{i-1}\frac{q_j}{|\vec{x}_i-\vec{x}_j|}$$

$$\boxed{W = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{n}\sum_{j=1}^{i-1}\frac{q_j q_i}{|\vec{x}_i-\vec{x}_j|} = \frac{1}{8\pi\epsilon_0}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{q_i q_j}{|\vec{x}_i-\vec{x}_j|}}$$



Madelung energies