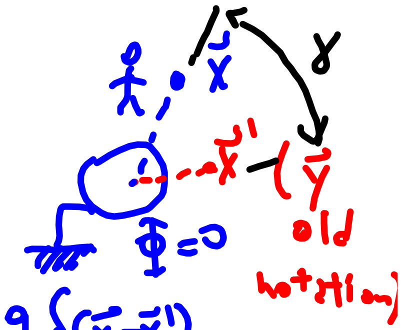
2.6 Green function of the sphere

$$G_D(\vec{x},\vec{x}') = \frac{1}{|\vec{x}-\vec{x}'|} + \underbrace{F(\vec{x},\vec{x}')}_{\nabla^2 F = 0}$$



$$\nabla^2 G_D = -4\pi\,\delta(\vec{x}-\vec{x}')$$

$G_D = 0$ at surface

$$\nabla^2 \Phi(\vec{x}) = -\frac{\rho}{\epsilon_0} = -\frac{q\,\delta(\vec{x}-\vec{x}')}{\epsilon_0}$$

special case and $q = 4\pi\epsilon_0$

$\Phi(\vec{x})\big|_{\text{surface}} = 0$

$$\boxed{G_D = \frac{1}{|\vec{x}-\vec{x}'|} + \text{contribution of the image charge}}$$

$$= \frac{1}{\sqrt{x^2+x'^2-2xx'\cos\gamma}} - \frac{1}{\sqrt{\frac{x^2x'^2}{a^2}+a^2-2xx'\cos\gamma}}$$

$\phi = \int_V \rho G_D + \oint_S \phi \frac{\partial G_D}{\partial n'}$

$G_D(\vec{x},\vec{x}') = G_D(\vec{x}',\vec{x})$

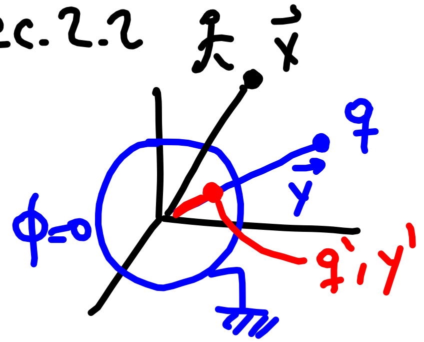
"old" y

$\nabla^2 \frac{1}{|\;\;|} = 0$

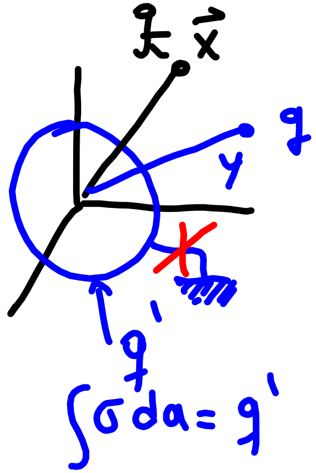
2.3 Point charge in the presence of a charged, insulated, conducting sphere.

↑ Q

Sec.2.2

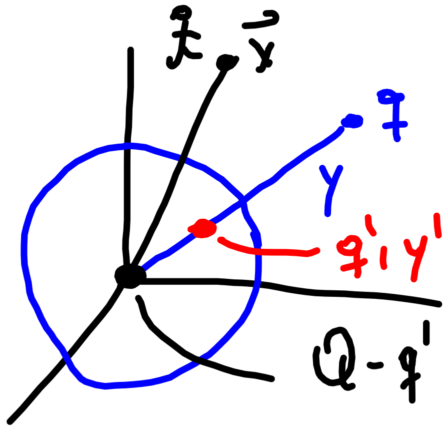


Equilibrium



First, connect to the ground without Q. Equilibrium.

Distribute $Q - q'$ on the sphere (uniform)



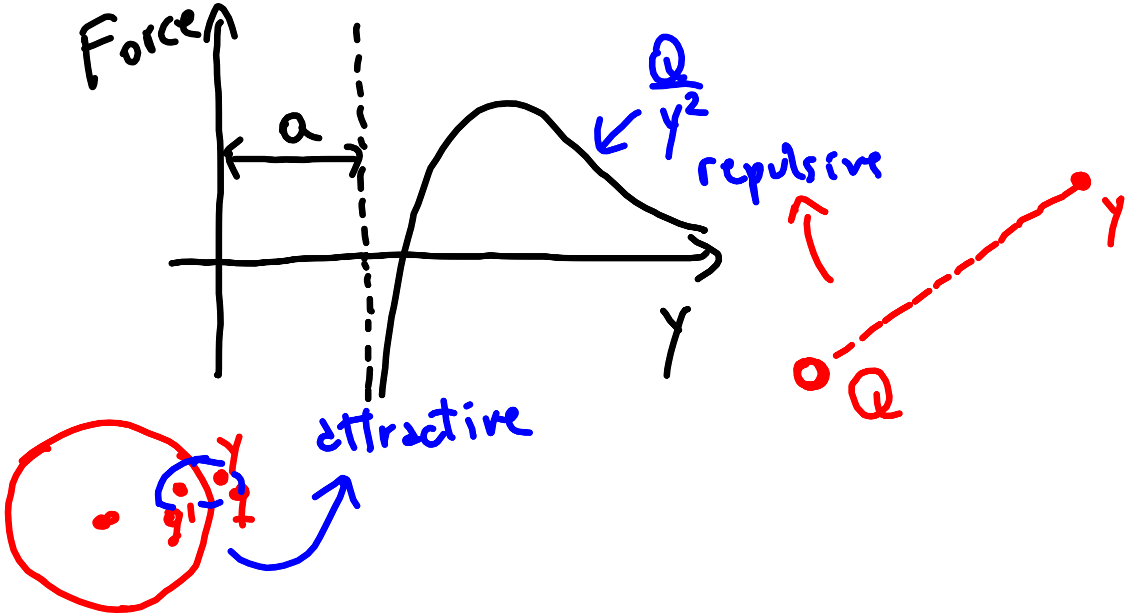
equivalent to 3 point like charges

$$\Phi(\vec{x}) = \frac{1}{4\pi\epsilon_0}\left[\frac{q}{|\vec{x}-\vec{y}|} + \frac{\left(-\frac{a}{y}q\right)}{\left|\vec{x}-\frac{a^2}{y^2}\vec{y}\right|} + \frac{Q-\left(-\frac{a}{y}q\right)}{|\vec{x}|}\right]$$

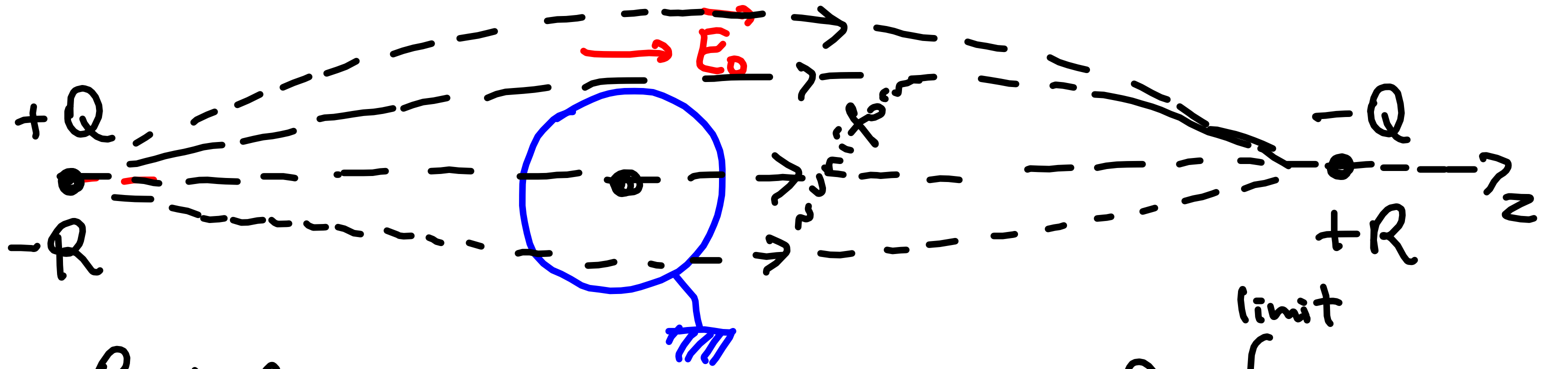
real; image; obs. point; location of q; location of q'; q'; charge at origin

Eq. 2.9 ; Force at q due to q' and Q-q'

Work function of metals



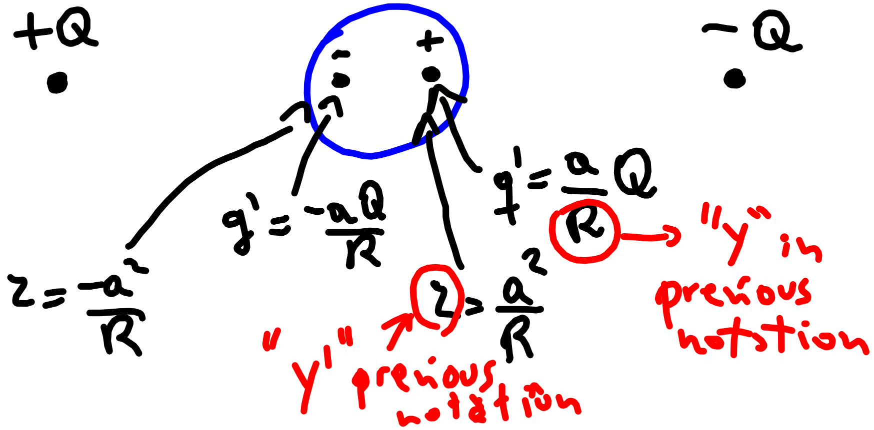
2.5 Conducting sphere in a uniform electric field



$R \to \infty$

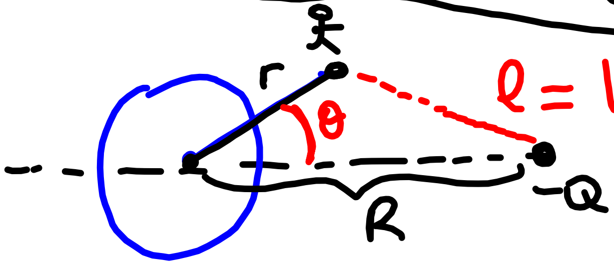
$Q \to \infty$

$$\frac{Q}{R^2} = \text{constant} = 2\pi\epsilon_0 E_0 \quad \Bigg\} \quad \underset{\text{at sphere}}{E} = 2 \cdot \frac{Q}{4\pi\epsilon_0 R^2} \overset{\text{limit}}{=} E_0$$



$$\Phi(\vec{x}) = \underbrace{\Phi_1 + \Phi_2 + \Phi_3 + \Phi_4}_{Q \qquad -Q \qquad q'_Q \qquad q'_{-Q}}$$ (2 real, 2 images)

Φ_2:

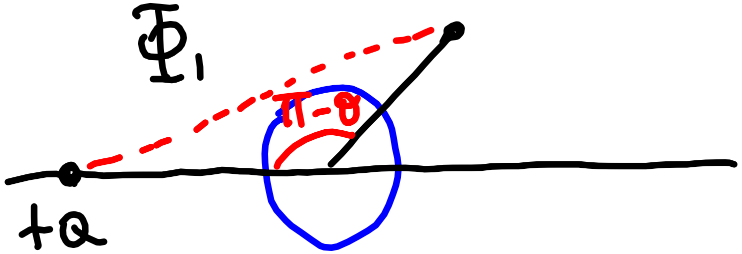


$$\ell = \sqrt{r^2 + R^2 - 2rR\cos\theta}$$

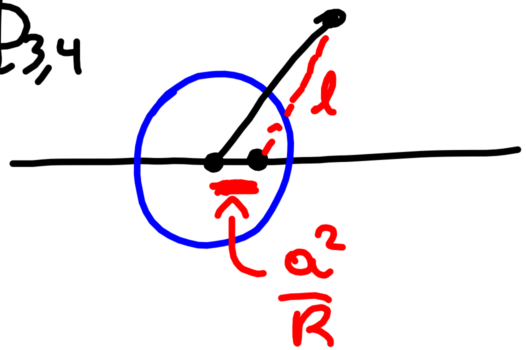
expand in $\frac{1}{R}$

$$\Phi_2 = -\frac{Q}{4\pi\epsilon_0}\frac{1}{\sqrt{r^2+R^2-2rR\cos\theta}} \approx -\frac{Q}{4\pi\epsilon_0}\frac{1}{R}\left[1 + \frac{r}{R}\cos\theta\right]$$

Φ_1



$\Phi_{3,4}$



$$\Phi = \frac{Q}{\underbrace{4\pi\epsilon_0 R^2}}\left[-2r\cos\theta + \frac{2a^3}{r^2}\cos\theta\right] + \cdots$$

$$\frac{Q}{R^2} = 2\pi\epsilon_0 E_0 \qquad Q \to \infty, \quad R^2 \to \infty$$

$$\Phi = -E_0\left[r\cos\theta - \frac{a^3}{r^2}\cos\theta\right]$$

constant $\vec{E_0}$

$r\cos\theta = z$

dipolar contribution

$$\boxed{E = E_0 + \frac{2E_0 a^3}{z^3}}$$

along the z axis