$$q_{10} = \sqrt{\frac{3}{4\pi}}\, p_z \qquad q_{1-1} = -q_{11}^{*}$$

$$q_{22} \stackrel{\text{def}}{=} \int \underbrace{Y_{22}^{*}(\theta',\phi')}_{\frac{1}{4}\sqrt{\frac{15}{2\pi}}\sin^2\theta' e^{-2i\phi'}} \underbrace{(r')^2}_{r'^2 \sin^2\theta' (\cos\phi' - i\sin\phi')^2} \rho(\vec{x}')\, d^3x'$$

$$= \frac{1}{12}\sqrt{\frac{15}{2\pi}} \int d^3x'\, \rho(\vec{x}')\, 3\,(x'^2 - 2i x' y' - y'^2)$$

$$\boxed{Q_{ij} = \int (3x_i' x_j' - r'^2 \delta_{ij})\, \rho(\vec{x}')\, d^3x'}$$

$x_1' = x',\ x_2' = y',\ x_3' = z'$

$$q_{22} = \frac{1}{12}\sqrt{\frac{15}{2\pi}}\left(Q_{11} - 2i\,Q_{12} - Q_{22}\right)$$

$$\frac{1}{2}\frac{(p_x - i p_y)(x+iy)}{r^3}$$

$$\Phi = \frac{1}{4\pi\epsilon_0}\sum_{l,m}\frac{4\pi}{2l+1}\, q_{lm}\,\frac{Y_{lm}}{r^{l+1}}$$

$$-\sqrt{\frac{3}{8\pi}}\left(p_x - i p_y\right)$$

$$-\sqrt{\frac{3}{8\pi}}\,\sin\theta\, e^{i\phi}$$

$$\frac{1}{\sqrt{4\pi}}$$

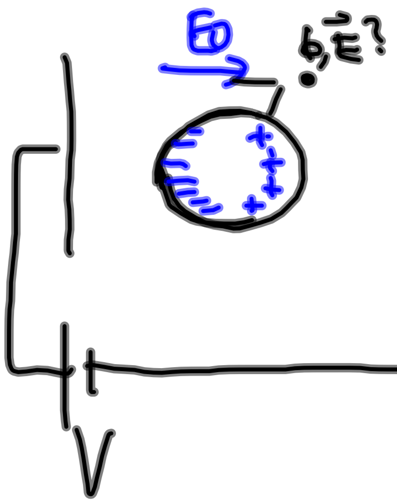
$$= \frac{1}{4\pi\epsilon_0}\Big[\underbrace{4\pi\, q_{00}\,\frac{Y_{00}}{r}}_{l=m=0} + \frac{4\pi}{3} q_{11}\frac{Y_{11}}{r^2} + \frac{4\pi}{3} q_{10}\frac{Y_{10}}{r^2} + \frac{4\pi}{3} q_{1-1}\frac{Y_{1-1}}{r^2} + \cdots\Big]$$

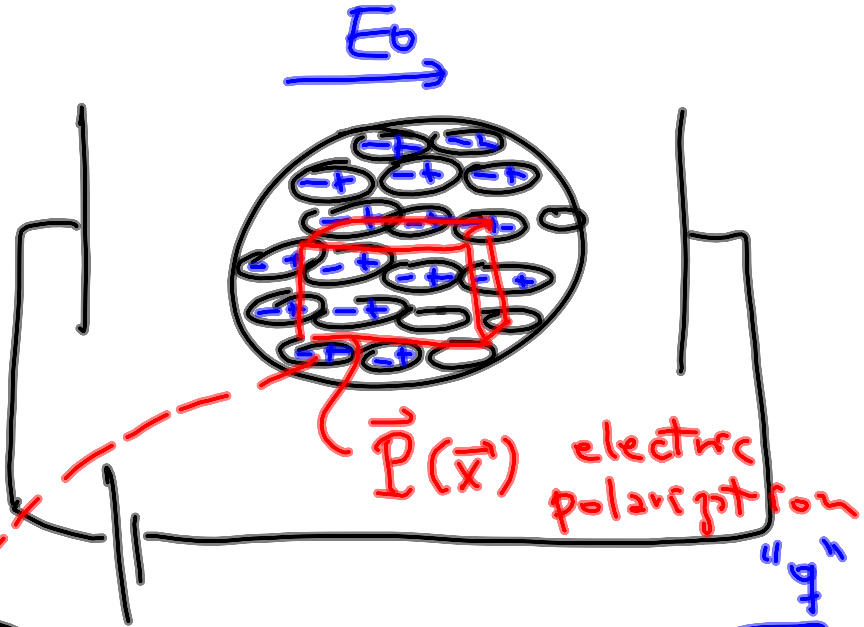
$$\frac{q}{\sqrt{4\pi}}$$

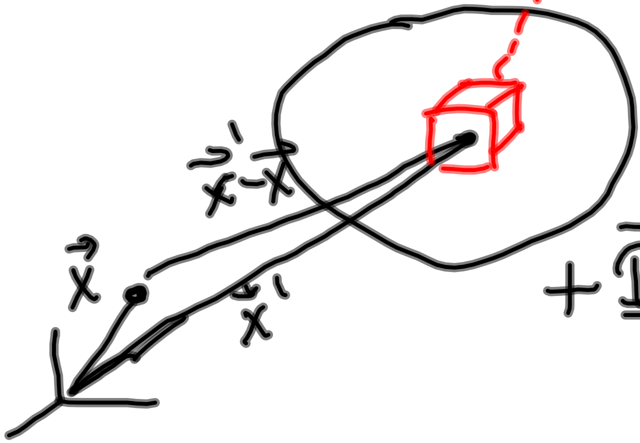
$$= \frac{1}{4\pi\epsilon_0}\left[\frac{q}{r} + \frac{\vec{p}\cdot\vec{x}}{r^3} + \frac{1}{2}\sum_{i,j} Q_{ij}\frac{x_i x_j}{r^5} + \cdots\right]$$

$$\vec{E} = -\nabla\Phi$$

4.3 Electrostatics with Ponderable Media







$$\Delta\Phi(\vec{x},\vec{x}') = \frac{1}{4\pi\epsilon_0}\left[\frac{\overbrace{\rho(\vec{x}')\Delta v}^{``q"}}{|\vec{x}-\vec{x}'|} + \frac{\vec{P}(\vec{x}')\cdot(\vec{x}-\vec{x}')\Delta v}{|\vec{x}-\vec{x}'|^3} + \dots\right]$$

$$\Phi(\vec{x}) \simeq \frac{1}{4\pi\epsilon_0}\int d^3x' \left[\frac{\rho(\vec{x}')}{|\vec{x}-\vec{x}'|} + \frac{\vec{P}(\vec{x}')\cdot(\vec{x}-\vec{x}')}{|\vec{x}-\vec{x}'|^3} \right]$$

$$\frac{(\vec{x}-\vec{x}')}{|\vec{x}-\vec{x}'|^3} \to \nabla'\left(\frac{1}{|\vec{x}-\vec{x}'|}\right)$$

integ. by parts

$$\int \nabla\cdot\left(\frac{1}{|\,|}\vec{P}\right) = 0$$

$$= \frac{1}{4\pi\epsilon_0}\int d^3x' \frac{1}{|\vec{x}-\vec{x}'|}\underbrace{\left[\rho(\vec{x}') - \nabla'\cdot\vec{P}(\vec{x}')\right]}_{\text{effective charge density}}$$

$$\nabla^2\Phi = \int \underbrace{\nabla^2\frac{1}{|\,|}}_{-4\pi\delta}[\quad] \qquad -\nabla\cdot\vec{E}$$

$$\boxed{\nabla^2\Phi = -\frac{1}{\epsilon_0}(\rho - \nabla\cdot\vec{P})}$$

$$\vec{D} \stackrel{\text{def}}{=} \epsilon_0\vec{E} + \vec{P} \quad \Big| \quad \nabla\cdot\vec{D} = \epsilon_0\nabla\cdot\vec{E} + \nabla\cdot\vec{P}$$

$$\boxed{\nabla\cdot\vec{D}=\rho}$$

$$\nabla^2\Phi = -\frac{1}{\epsilon_0}\left(\rho-\nabla\cdot\vec{P}\right)$$

$$\vec{D}=\epsilon_0\vec{E}+\vec{P}$$

$$\vec{P}=\epsilon_0\chi_e\vec{E}$$

χ_e: electric susceptibility of the medium

$$\vec{D}=\epsilon_0\vec{E}+\epsilon_0\chi_e\vec{E}$$

$$=\underbrace{\epsilon_0(1+\chi_e)}_{\epsilon}\vec{E}$$

ϵ: electric permitivity

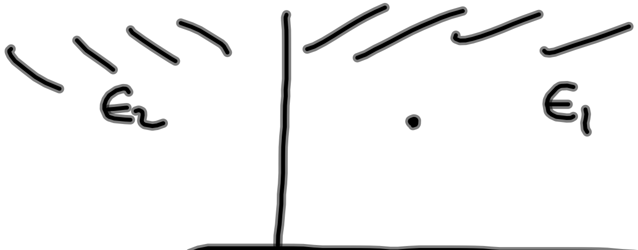
$$\text{dielectric constant}=\frac{\epsilon}{\epsilon_0}$$

χ_e is uniform in space

$$\nabla\cdot\vec{E} = \frac{1}{\epsilon_0}\left(\rho - \nabla\cdot\vec{P}\right) = \frac{1}{\epsilon_0}\left(\rho - \epsilon_0\chi_e\nabla\cdot\vec{E}\right)$$

χ_e unif. in space

$$\boxed{\nabla\cdot\vec{E} = \frac{\rho}{\epsilon_0(1+\chi_e)} = \frac{\rho}{\epsilon}}$$



$$\boxed{\left(\vec{D}_2 - \vec{D}_1\right)\cdot\vec{n}_{21} = \sigma}$$

true charge

$$\boxed{\left(\vec{E}_2 - \vec{E}_1\right)\times\vec{n}_{21} = 0}$$

$$\nabla\times\vec{E} = 0$$