First Midterm Exam

P555

February 13, 2020

SHOW ALL WORK TO GET FULL CREDIT!

Problem 1: Consider the array of points shown in Fig. 1.

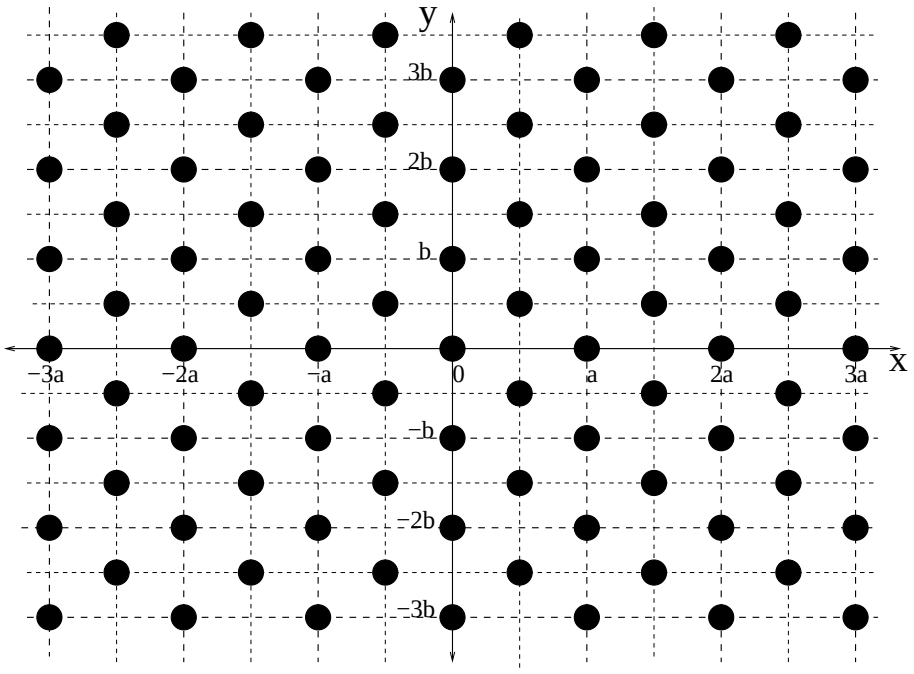

FIG. 1:

a) Explain why the points form a Bravais lattice and name (provide the name of) the lattice.(5 points)

b) Provide a set of primitive vectors for this lattice. Draw the vectors in Fig. 1 and write an expression for them in cartesian coordinates in terms of the constants a and b.(5 points)

c) Find the primitive vectors for the reciprocal lattice of the Bravais lattice shown in Fig. 1. Provide an expression for them in cartesian coordinates in terms of the constants a and b. (5 points)

d) Draw the vectors found in part (c) in Fig. 2 and identify (provide the name of) the Bravais lattice that they generate.(5 points)

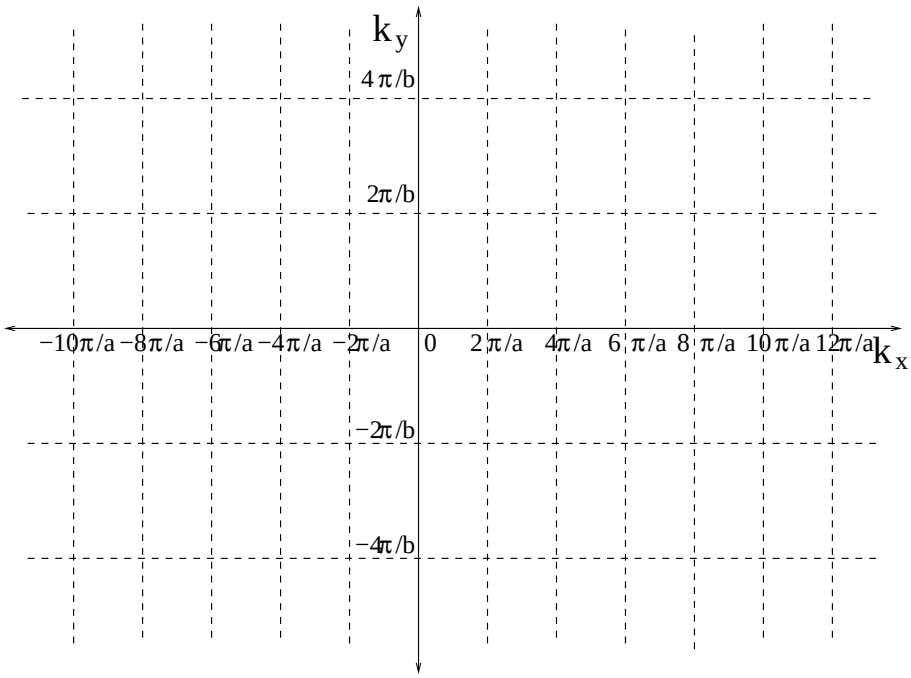

FIG. 2:

e) In cartesian coordinates provide an expression for a generic vector **K** in reciprocal space and indicate with a circle in Fig. 2 the end point of all the reciprocal vectors that fit in the figure (Hint: verify that the points you draw agree with the answer you provided in (d)). (5 points)

Problem 2: Consider the array of atoms shown in Fig. 3.

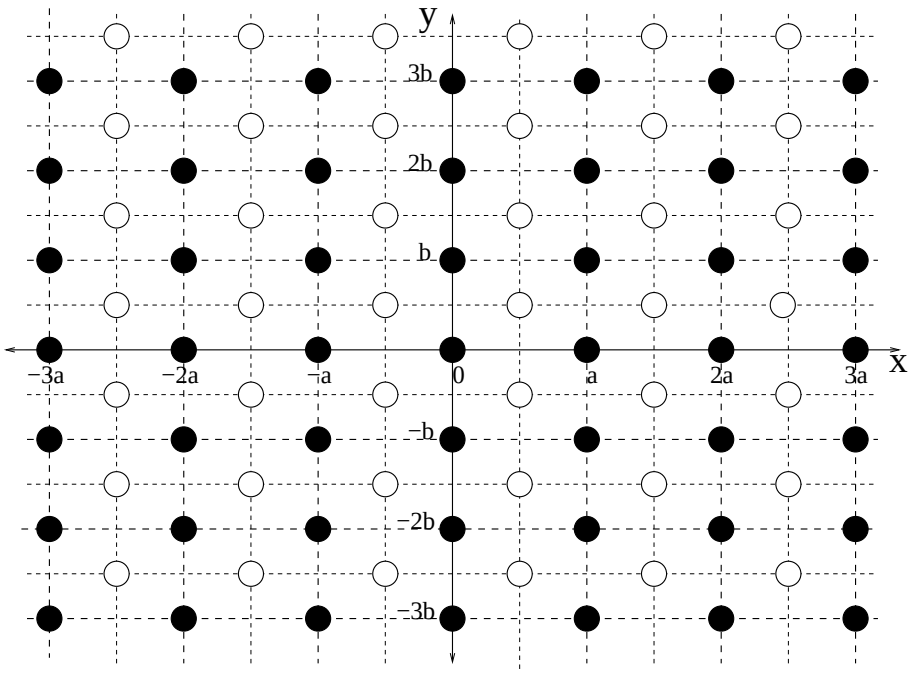

FIG. 3:

a) Are all the atoms in the figure sitting at the sites of a Bravais lattice or is this system represented by a Bravais lattice with a basis? Justify your answer. (5 points)

b) Name (provide the name of) the Bravais lattice and, if needed, provide a set of basis vectors (in cartesian coordinates and in terms of a and b) and draw the basis in Fig. 3. (5 points)

c) Provide a set of primitive vectors for the Bravais lattice. Draw the vectors in Fig. 3 and write an expression for them in cartesian coordinates in terms of the constants a and b. (5 points)

d) Find the primitive vectors for the reciprocal lattice of the Bravais lattice shown in Fig. 3. Provide an expression for them in cartesian coordinates in terms of the constants a and b. (5 points)

e) Draw the vectors found in part (d) in Fig. 4 and identify (provide the name of) the Bravais lattice that they generate. (5 points)

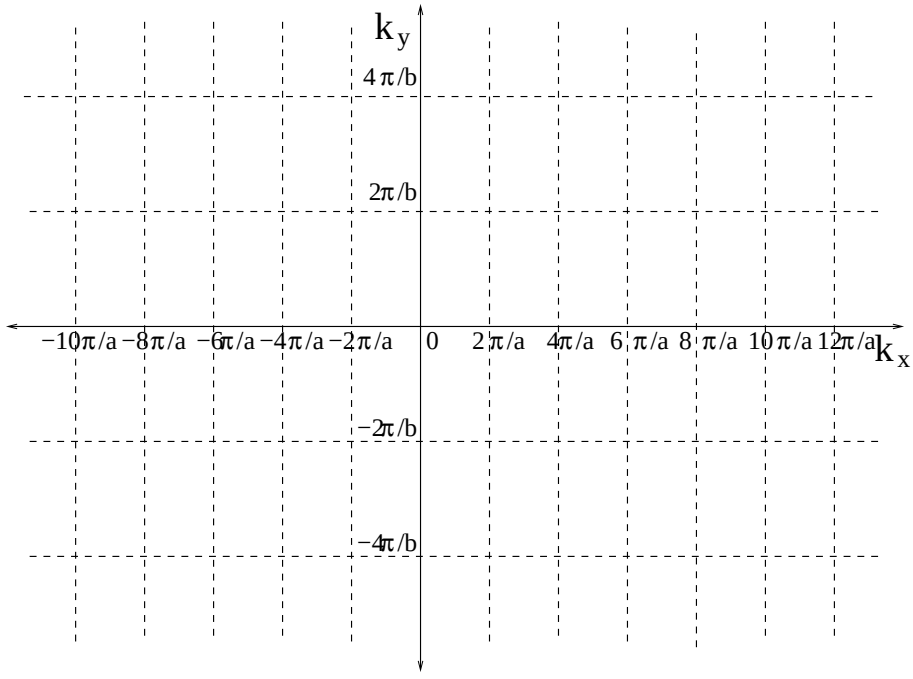

FIG. 4:

f) In cartesian coordinates provide an expression for a generic vector $\mathbf{K}$ in reciprocal space and indicate with a circle in Fig. 4 the end point of all the reciprocal vectors that fit in the figure (Hint: verify that the points you draw agree with the answer you provided in (e)). (5 points)

g) Now assume that the white atoms in Fig. 3 are replaced by black atoms, i.e., all the atoms in the figure are now identical. Calculate the modulation factor $F_{\mathbf{k}}$. (5 points)

h) Find an expression for the zeroes in $F_{\mathbf{k}}$. (5 points)

i) In Fig. 4 draw a circle around the points in reciprocal space for which $F_{\mathbf{k}} = 0$. (5 points)

j) Compare the pattern you obtained in Fig. 4 with the points in Fig. 2. What would you expect? Does the result you found match your expectations? (5 points)