P555

May 6, 2021

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Problem 1: In the periodic table we see that the P atom has an electronic structure given by [Ne]$3s^23p^3$.

a) Use Hund rules to obtain S, L, and J for the ground state of the P atom. Draw the energy levels in the relevant shells and indicate the electronic placement. Provide your final result using spectroscopic notation: ${}^{2S+1}L_J$. (5 points)

b) Provide the electronic structure of the phosphorus ion P^+. (5 points)

c) Use Hund rules to obtain S, L, and J for the ground state of the P^+ ion. Draw the energy levels in the relevant shells and indicate the electronic placement. Provide your final result using spectroscopic notation: ${}^{2S+1}L_J$. (5 points)

d) Now provide the electronic structure of the P ion P^-. (5 points)

e) Use Hund rules to obtain S, L, and J for the ground state of the P^- ion. Draw the energy levels in the relevant shells and indicate the electronic placement. Provide your final result using spectroscopic notation: ${}^{2S+1}L_J$. (5 points)

Problem 2: Consider a two-dimensional rectangular lattice of particles with mass M and lattice constants a and $b = a/2$. Let $\hat{r}_{ij}$ be a unit vector pointing from the equilibrium location $\mathbf{R}_i$ of particle i to the equilibrium location $\mathbf{R}_j$ of particle j. Let $\mathbf{u}_i$ give the two dimensional displacement of particle i from its equilibrium location. Suppose that there is a nearest neighbor harmonic potential between the atoms. The spring constant along a is K_a and along b is K_b with $K_b = K_a/4$.

a) Provide a set of primitive vectors for the lattice. (5 points)

b) Provide the number n of nearest neighbors for an atom located at site $\mathbf{R}_i$ and provide the location $\mathbf{r}_{i,j}$ of each of the neighbors (with $j = 1, ..., n$) in terms of the primitive vectors that you provided in (a). (5 points)

c) Find the two equations in two unknowns whose solution would give the dispersion relation $\omega_{\nu\mathbf{k}}$ for vibrations of the lattice. (5 points)

d) Plot the two solutions $\omega_{1\mathbf{k}}$ and $\omega_{2\mathbf{k}}$ versus $\mathbf{k}$ along the path in k-space $Y - \Gamma - X$ where $\Gamma = (k_x, k_y) = (0, 0)$, $X = (k_x, k_y) = (\pi/a, 0)$, and $Y = (k_x, k_y) = (0, \pi/b)$. Use a different color for $\omega_{1\mathbf{k}}$ and $\omega_{2\mathbf{k}}$ and in each panel of the plot identify which of the two is the longitudinal mode and which one is the transverse mode. (5 points)

e) Take the limit $k \to 0$ and find the speed of sound along a and along b in this system. Along what direction is the speed of sound larger? (5 points)