First Midterm Exam

P555

March 3, 2022

SHOW ALL WORK TO GET FULL CREDIT!

Problem 1: Consider the array of atoms shown in Fig. 1.

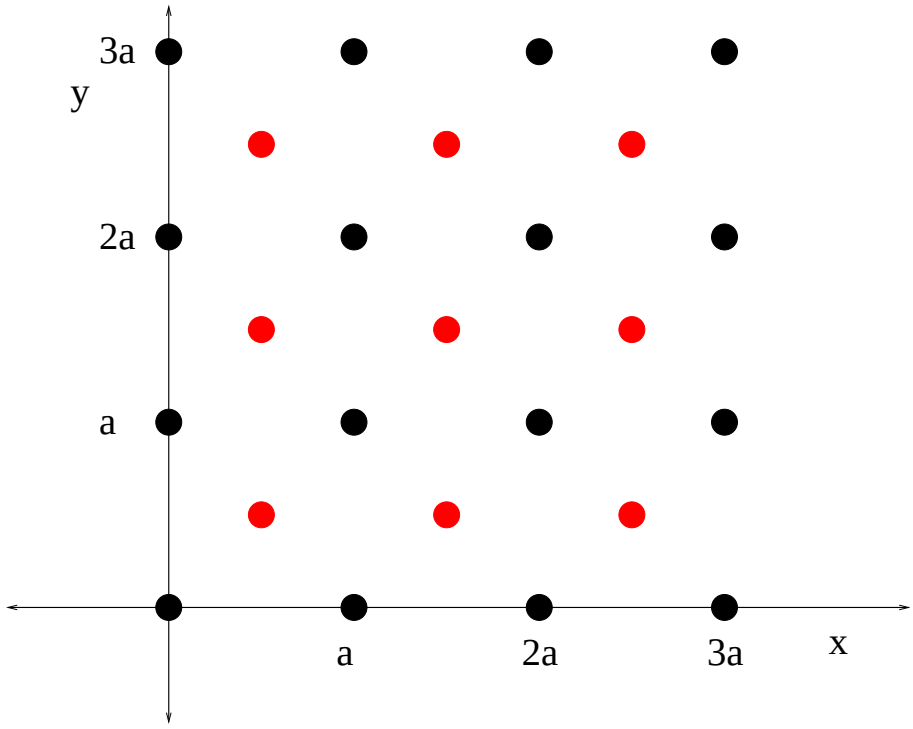

FIG. 1:

a) What is the Bravais lattice?(5 points)

b) Provide an expression for the primitive vectors $\mathbf{a}_1$ and $\mathbf{a}_2$ in cartesian coordinates in terms of the distance a indicated in Fig. 1 and draw the vectors in the figure. (5 points)

c) Draw a primitive unit cell in Fig. 1. (5 points)

d) How many atoms are in the primitive unit cell that you drew? (5 points)

e) How many points of the Bravais lattice are in the primitive unit cell that you drew? (5 points)

f) Do you need to provide a basis to represent the structure in Fig. 1? If you need a basis provide an expression for the basis vectors in terms of the distance a indicated in the figure.(5 points)

g) Find the primitive vectors $\mathbf{b}_1$ and $\mathbf{b}_2$ in the reciprocal lattice. (5 points)

h) Now the red atoms are replaced by black atoms so that all the atoms in the material are the same as shown in Fig. 2. Calculate the modulation factor $F_{\mathbf{K}}$ if you provided a basis in part (f). (5 points)

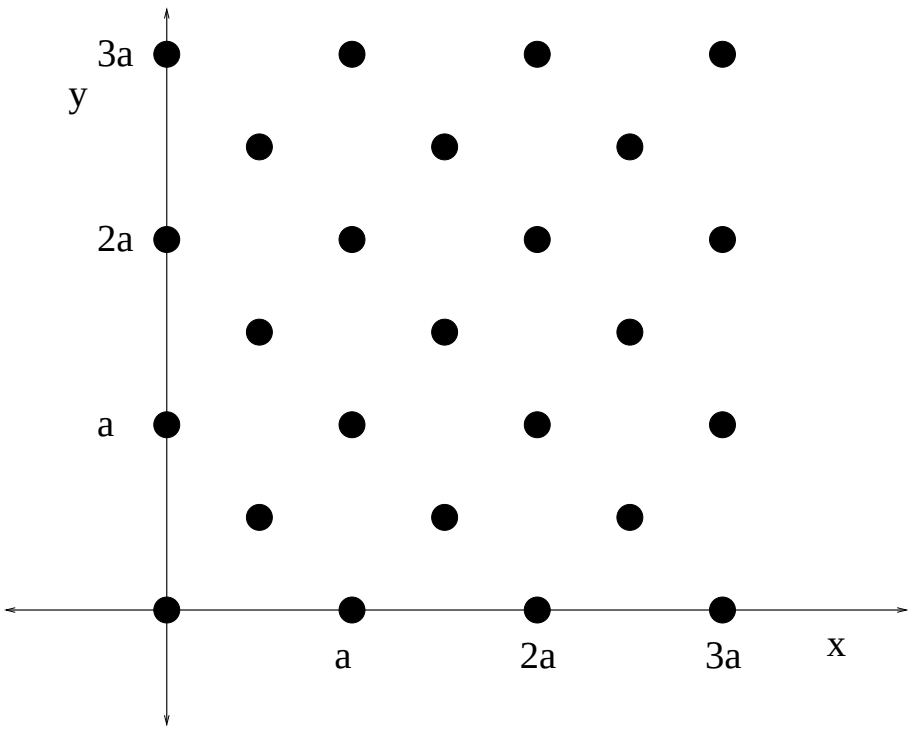

FIG. 2:

i) Under what conditions $F_{\mathbf{K}} = 0$? Provide a vector of the reciprocal lattice for which the modulation factor vanishes.(5 points)

j) For the lattice shown in Fig. 2 with all the atoms being the same, draw a primitive unit cell in Fig. 2.(5 points)

k) Provide an expression for the primitive vectors $\mathbf{a'}_1$ and $\mathbf{a'}_2$ in cartesian coordinates in terms of the distance a indicated in Fig. 2 and name the Bravais lattice. (5 points)

l) How many atoms are in the primitive unit cell? (5 points)

m) Does this lattice need a basis? If the answer is yes, provide the basis (5 points)

Problem 2: Consider a free Fermi gas in 2 dimensions with N electrons, i.e., the electrons are free inside a square box of side L. Assume periodic boundary conditions.

a) Provide the values of the momentum $\mathbf{k}$ that one single electron can have. (5 points)

b) Provide an expression for the energy that one single electron can have. (5 points)

c) Provide the energy of the 4 lowest energy levels and indicate the degeneracy of each level. (5 points)

d) Find the energy of the ground state when there are N=20 electrons inside the box. (5 points)