First Midterm Exam

P555
March 2, 2023

SHOW ALL WORK TO GET FULL CREDIT!

Problem 1: The array of atoms shown in Fig. 1 describe a square lattice with lattice constant a.

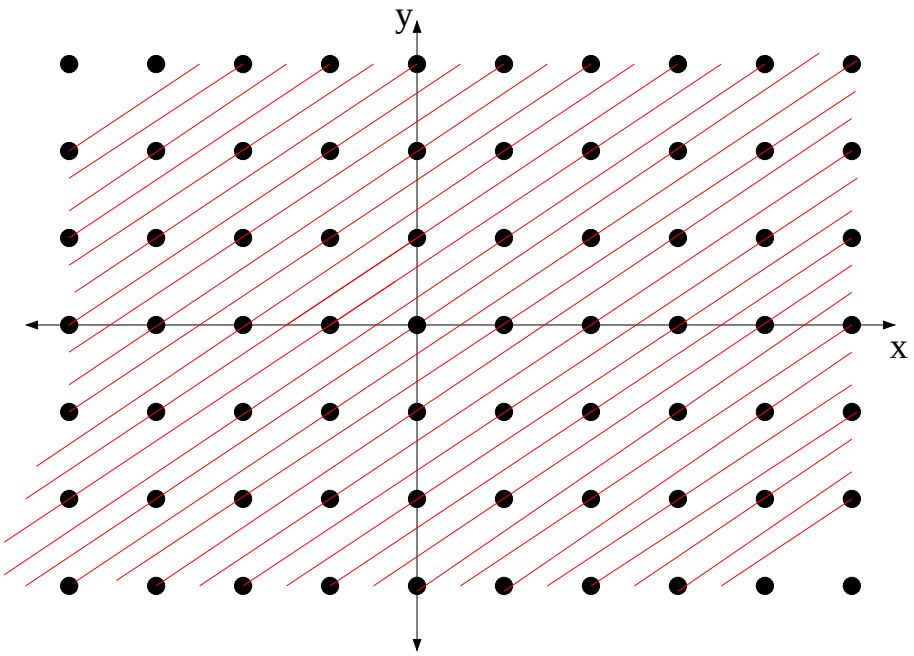

FIG. 1:

a) Provide an expression for a set of primitive vectors $\mathbf{a}_1$ and $\mathbf{a}_2$ in cartesian coordinates in terms of the lattice constant a and draw the vectors in Fig. 1. (5 points)

b) A family of planes is indicated with red lines in Fig. 1. Provide the Miller indices for this family of planes and explain how you obtained them. (5 points)

c) Write an expression for the vector $\mathbf{K}$ in reciprocal space which is perpendicular to the family of planes in Fig. 1. (5 points)

d) What is the distance d between the planes? Explain your reasoning. (5 points)

e) Now assume that x-rays with $\mathbf{k}_0 = \frac{2\pi}{a}(\frac{13}{4}, 0)$ are shone on the family of red planes. Provide the value $\mathbf{k}$, a vector, that the scaterred radiation will have. Show your work. (5 points)

g) In Fig. 2 the points in the reciprocal space of Fig. 1 are shown. What is the separation between the nearest neighbor points in the figure? Provide your answer in terms of a. (5 points)

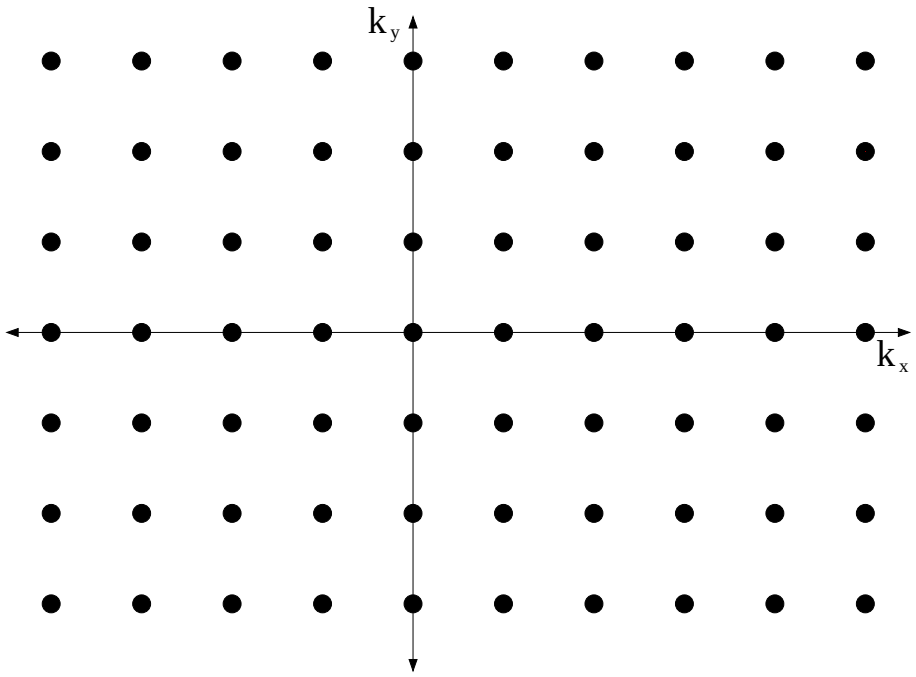

FIG. 2:

h) Provide an expression for a set of primitive vectors $\mathbf{b}_1$ and $\mathbf{b}_2$ for the reciprocal lattice shown in Fig. 2 in cartesian coordinates and draw the vectors in Fig. 2. (5 points)

i) Now draw in Fig. 2 the vector $\mathbf{K}$ that you found in part (c), the vector $\mathbf{k}$ you found in part (e) and vector $\mathbf{k}_0$ provided in (e) showing that the conditions for constructive interference are satisfied by these vectors. (5 points)

Problem 2: Consider a free Fermi gas in 3 dimensions with N electrons, i.e., the electrons are free inside a cubic box of side L. Assume periodic boundary conditions.

a) Provide the values of the momentum $\mathbf{k}$ that one single electron can have. (5 points)

b) Provide an expression for the energy that one single electron can have. (5 points)

c) Provide the energy of the 4 lowest energy levels and indicate the degeneracy of each level. (5 points)

d) Find the energy of the ground state when there are N=20 electrons inside the box. (5 points)

e) Find the Fermi energy of the system. (5 points)