First Midterm Exam

P555

February 29, 2024

SHOW ALL WORK TO GET FULL CREDIT!

Problem 1: Fig. 1 shows an array of two different kinds of atoms in real space: black and red. The axis of coordinates are shown.

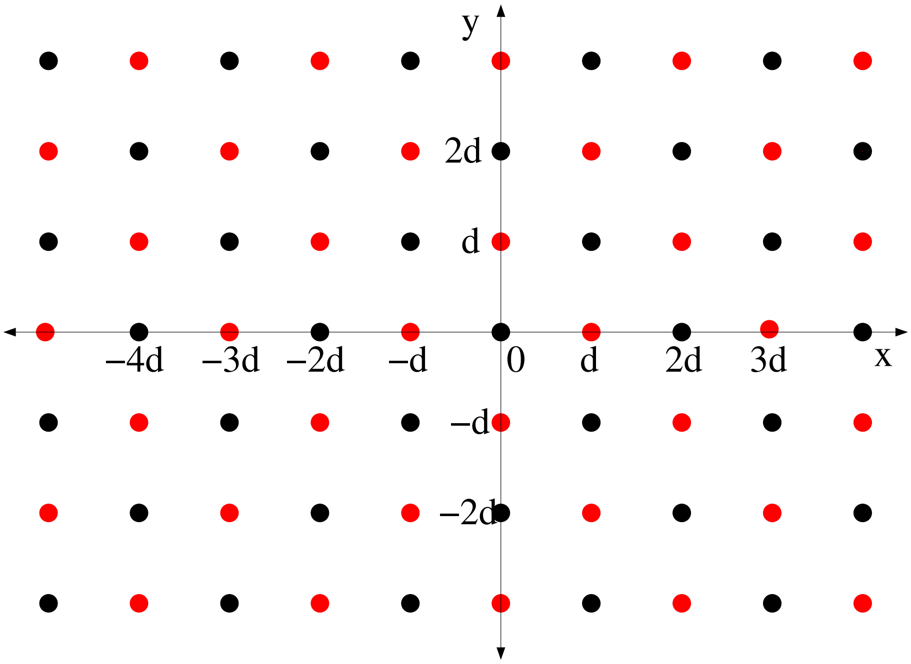

FIG. 1:

a) What is the two dimensional Bravais lattice in which the atoms in Fig. 1 are arranged? Provide the lattice constants in terms of the distance d. (5 points)

b) Provide an expression for a set of primitive vectors $\mathbf{a}_1$ and $\mathbf{a}_2$ in cartesian coordinates in terms of the length d shown in the figure and draw the vectors in Fig. 1. (10 points)

c) Draw a primitive unit cell and indicate how many atoms and how many points of the Bravais lattice are inside the cell. Justify your answer. (5 points)

d) Explan why a basis is needed to describe the array of atoms shown in Fig. 1, provide an expression for the basis vectors, and draw them in the figure. (10 points)

e) Find a basis for the reciprocal lattice corresponding to the Bravais lattice that you found in part (b). Draw the basis vectors in Fig. 2 and label each tick mark in Fig. 2 in terms of the length d provided in Fig. 1. The dashed lines in Fig. 2 are provided to help you in your drawings.(10 points)

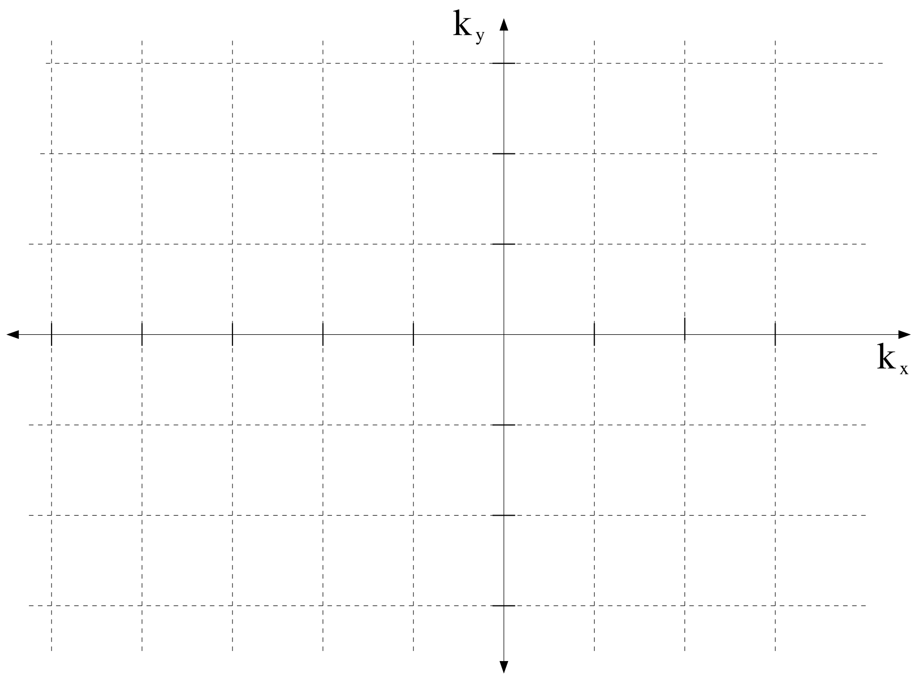

FIG. 2:

f) Write an expression for a generic vector $\mathbf{K}$ in reciprocal space and indicate with black dots in Fig. 2 all the vectors of the reciprocal lattice that fit in the figure. The dashed lines in Fig. 2 are provided to help you in your drawings. (10 points)

g) Now assume that the red atoms in Fig. 1 are identical to the black atoms and provide an expression for the modulation factor $F_{\mathbf{K}}$. (5 points)

h) Are there points in the diffraction pattern that you drew in Fig. 2 that will vanish? If this is the case indicate which points will vanish and draw in Fig. 3 the points that survive. (10 points)

Bonus) What does the difraction pattern that you drew in Fig. 3 indicate about the Bravais lattice in Fig. 1 when the red atoms are replaced by black atoms? Explain. Hint: think about what is the Bravais lattice in this situation, and what are the primitive vectors. (10 points)

Problem 2: Consider a free Fermi gas in 3 dimensions with N electrons. The electrons are free inside a box with a square bottom of side L and a height $2L$. Assume periodic boundary conditions.

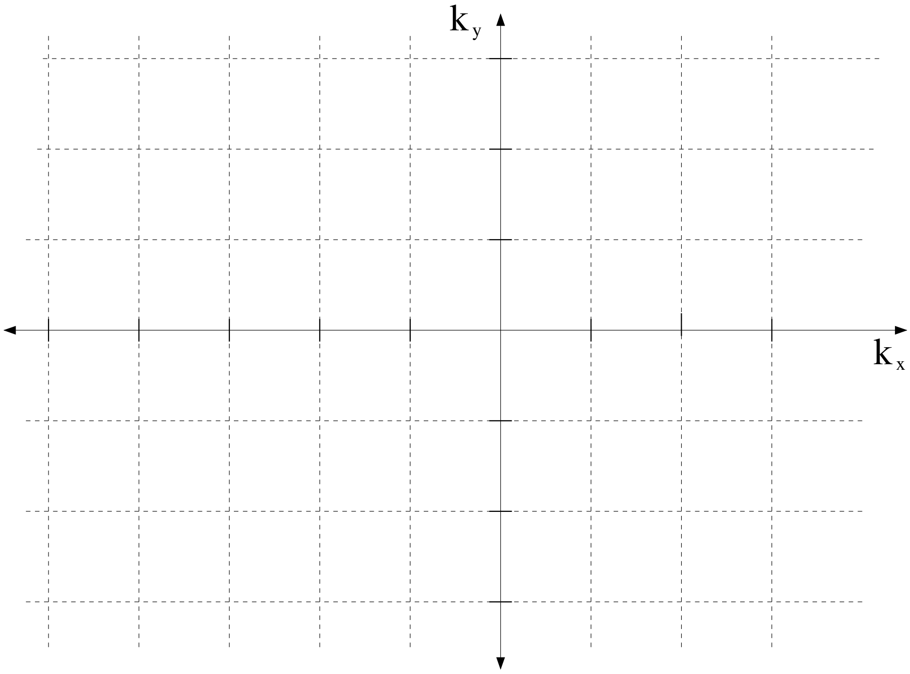

FIG. 3:

a) Provide the values of the momentum **k** that one single electron can have. (5 points)

b) Provide an expression for the energy that one single electron can have. (5 points)

c) Provide the energy of the 4 lowest energy levels and indicate the degeneracy of each level and make a figure indicating each of the 4 levels and its degeneracy. (10 points)

d) Find the energy of the ground state when there are N=10 electrons inside the box. Place the 10 electrons in the energy levels that you drew in part (c). (10 points)

e) Find the Fermi energy of the system. (5 points)