Gaussian Distribution and their Application :

Muscle Fiber Distribution on Cross-Section Muscle Area

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From a distribution of people's heights in a group to error signal detection, the Gaussian Distribution provides various applications to predict the probability of a generalized distribution. Especially, the Gaussian Distribution is generally applicable to explain a biological or physiological phenomenon because it shows the most realistic distribution in biology and physiology. In the human muscle, there are hundreds of muscle fibers to compose each muscle part. Depending on the location of each muscle fiber, it generates a different action potential to compose of the Electromyogram (EMG). Because of this fact, the locations of different muscle fibers are important. It will be shown how the location of each muscle fiber, based on the Gaussian Distribution, affects the final calculated muscle fiber action potential in this paper.

I. INTRODUCTION

The Gaussian distribution, also called Normal distribution, is the most applicable frequency distribution curve to real data analysis. From a simple mathematical calculation in probability to a complex approach to the *Uncertainty Principle* [1], the Gaussian distribution provides good application to solve problems. Initially, it was observed in measurement error distribution, and the original theorist, *Garl Friedrich Gauss*, also emphasized the error distribution when he addressed this distribution. This is the reason that this specialized distribution is called Gaussian. A Gaussian distribution of a population with mean value, m, and standard deviation, σ, can be expressed as;

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x-m)^2}{2\sigma^2}} \tag{1}$$

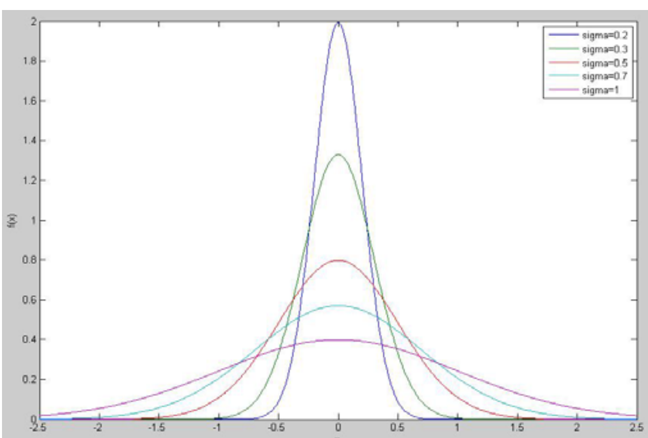

FIG. 1. Gaussian Distributions at a fixed mean, 0, with different sigmas(σ)

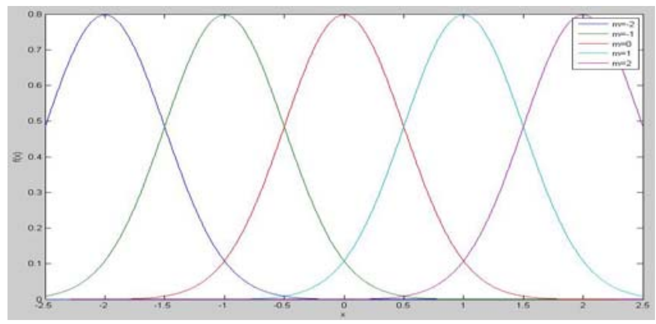

FIG. 2. Gaussian Distributions at a fixed sigma(σ), 0.5, with different mean values

Especially, the special case, m=0 and σ =1, is called as the Standard Normal distribution, and the numerical values based on this shows the specific values for the Gaussian distribution without additional calculation.

$$s(x) = \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}} \tag{2}$$

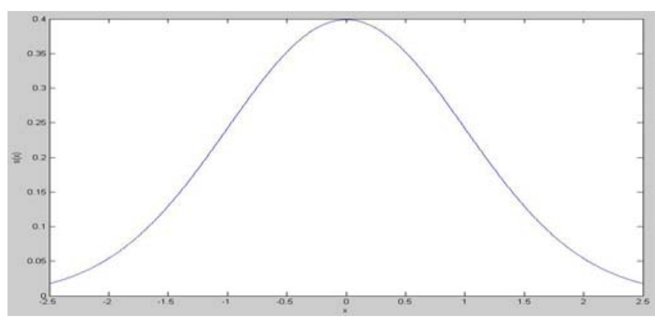

FIG. 3. Gaussian Distributions at sigma(σ)=1 & mean=0

This kind of distribution is easily applicable to physiological or biological phenomena. In the viscoelastic Fitzhugh-Nagumo models, for example, the density plot of membrane potential and its plane from the human heart indicates that there is Gaussian distribution relationship between action potential and the cardiac muscle fiber contraction [2]. The action potential under Gaussian distribution for insulted fiber in the cardiac muscles successfully described the physiological experiments. It can be also true in other physiological case, like muscle fiber distribution on the dissected area of the muscle. The skeletal muscle in the human body is composed of hundreds of fine muscle fibers. Each muscle fiber contracts and relaxes depending on action potential generated from the brain [3]. These action potentials are summed, and become the basis of Electromyogram (EMG), which is normally used for diagnosis in hospital or muscle examination for research. Because of the importance of EMG, the action potential in muscle fiber is also very important to study the human skeletal muscles. To simulate the action potential, there are several important factors to decide the shape of action potential; the diameter of fiber, the velocity of action potential flow, the location of fibers in the muscle, and muscle fiber types. Of these factors, the position of each muscle fiber can be considered as the most important factor because the different location of muscle fiber results in different muscle fiber action potential when it is detected on the surface of the skin. The differences can be the amplitudes of action potentials or the lasting time before converging to zero. According to Raja Dahmane, a professor in University of college of Health studies at Slovenia, the numbers of muscle fibers located under deeper area are more than those under surface [4]. Additionally, there are, as mentioned above, different types of muscle fiber; type I and type II. These different types of muscle fibers have different characteristics, and their population in the surface or deep location in the muscle also affects the EMG [4]. In this paper, a hypothetical model for the locations of muscle fibers is proposed by using the Gaussian distribution, and shows the different results from the effects with and without applying the Gaussian distribution.

II. METHODOLOGY

Based on previous researches [5, 6], a modified function to generate motor unit action potential was created.

$$V_m = 96(\lambda l)^3 e^{-\lambda l} - 90 \tag{3}$$

The "λ" is called Transmembrane coefficient, and obtained from experiment. Normally, it is set to 1. "*l*" is the traveling distance of action potential in each muscle fiber. Based on the muscle shape, one desired muscle model was created in Cartesian Coordinates. Originally, the model starts only 2-dimensional shape by using z-axis and y-axis. Its mathematical expression was an exponential equation like below;

$$y(z) = \alpha e^{-\frac{z^2}{0.17}} \tag{4}$$

The "*α*" was determined by the radius of the muscle, and the value of 0.17 was set for calculation. The function was applied for simulation to generate the volume conductor impulse response by using the impulse response function.

$$h = \frac{1}{\sqrt{5.24 \times \{(x_s - x_e)^2 + (y_s - y_e)^2 + (z_s - z_e)^2}} \tag{5}$$

The "*h*" is called volume conductor transfer function, and the convolution of the transmembrane current and this transfer function produce the extracellular potential, which is measured value on the skin. The (x_e, y_e, z_e) is the point value for detection, and the (x_s, y_s, z_s) is the point in the designed muscle model.

Before applying the Gaussian distribution for muscle fiber location, a uniformly distribution for muscle fibers was created in a normalized square area, which will be one quarter of the cross-sectional muscle area. Even though the shape of dissected muscle looks like a circle, it is assumed that the created area would be overlapped on the dissected muscle area. There are some muscle fibers which are out of the cross-sectional area when overlapped, but those numbers can be ignored because they are very small. The uniformly distributed muscle fiber model was converted into Gaussian distributed model by using *Box Müller transformation*, which is shown in (4) and (5). [7]

$$y_{BoxMuller} = \frac{\sqrt{-2\log(y)} \times \cos(2\pi z)}{4} \tag{6}$$

$$z_{BoxMuller} = \frac{\sqrt{-2\log(y)} \times \sin(2\pi z)}{4} \tag{7}$$

The *y* and *z* were original positions of muscle model, and they were divided by 4 to make a desired limited area. Several locations of muscle fibers under the uniformly distribution were chosen to generate their action potentials. At the same time, the action potentials at the locations by Gaussian distribution were also generated and compared with the previous results. Those selected locations are listed in Table I.

III. RESULTS

A. Muscle Fiber Distribution Pool

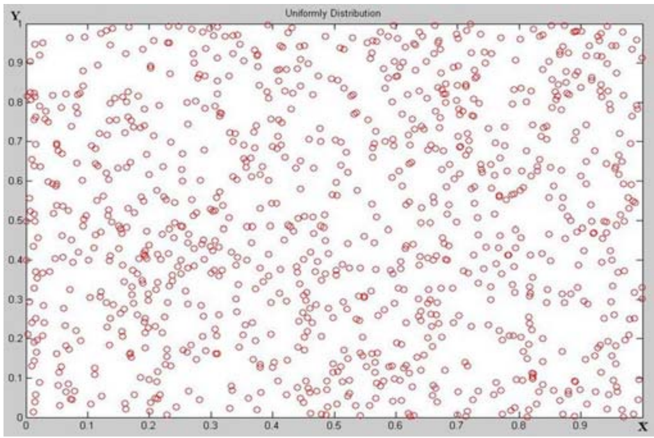

FIG. 4. The Pool model of Uniformly Distributed Muscle Fiber in a Quarter Area of Cross-Sectional Muscle Area

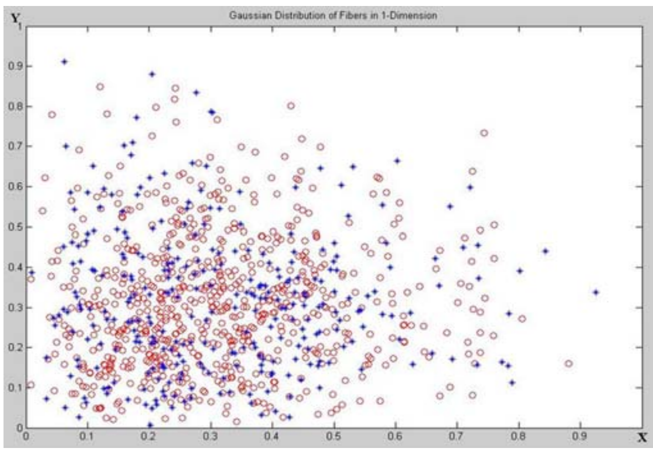

FIG. 5. The Pool model of Gassian Distributed Muscle Fiber in a Quarter Area of Cross-Sectional Muscle Area

In Fig.5, the red spots mean the locations of type I muscle fibers, and the blue spots are the locations of type II muscle fibers.

B. Muscle Fiber Action Potential

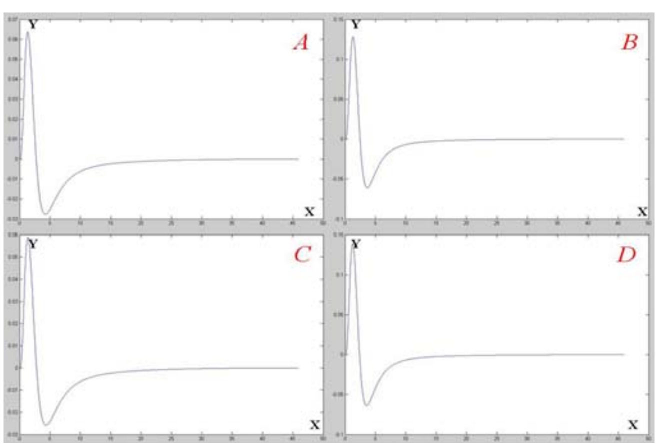

FIG. 6. Action Potentials from Fibers at different position. ***A*** is case 1 before applying the Gaussian Distribution, and ***B*** is case 1 after applying the Gaussian Distribution. As the same way, ***C*** is case 2 before applying the Gaussian Distribution, and ***D*** is case 2 after applying the Gaussian Distribution. The amplitudes in the figures are in [μV].

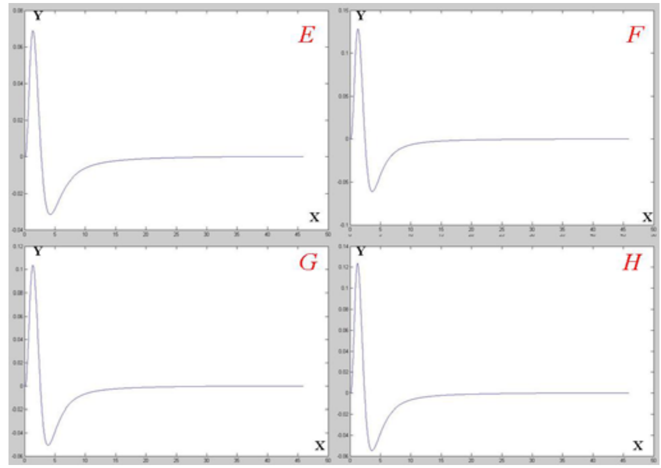

FIG. 7. Action Potentials from Fibers at different position. ***E*** is case 3 before applying the Gaussian Distribution, and ***F*** is case 3 after applying the Gaussian Distribution. ***G*** is case4 before, and ***H*** is case4 after applying Gaussian Distribution. The amplitudes in the figures are in [μV].

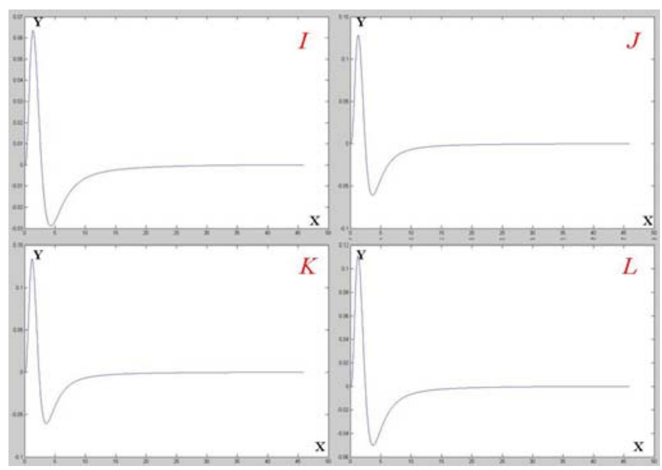

FIG. 8. Action Potentials from Fibers at different position. ***I*** is case 5 before applying the Gaussian Distribution, and ***J*** is case 5 after applying the Gaussian Distribution. ***K*** is case 6 before, and ***L*** is case 6 after applying the Gaussian Distribution. The amplitudes in the figures are in [μV].

All amplitudes in action potentials after applying the Gaussian distribution have increased. The selected cases showed increase in amplitudes after changed locations of muscle fiber. Except the SELECT 5 in Table 1, all amplitudes almost doubled after changing their positions. In the ***Table I***, the specific locations of chosen muscle fibers are addressed before and after applying the Gaussian distribution. The position (x, y) shows the location of muscle fiber in the uniformly distributed area, and the position (x_{Box}, y_{Box}) is the location of muscle fiber in

Gaussian distributed area. The boxmüller transfer function allows to convert the position under the uniformly distribution into the position under the Gaussian distribution. Based on these locations, the action potentials are generated. The designed cross-sectional muscle area was assumed as a circle with a radius of 3.5 [㎝].

TABLE I. Selected Muscle Fiber Locations : before (x, y) & after (x_{Box}, y_{Box}) Gaussian Distribution

	(x, y)		→	(x_{Box}, y_{Box})	
SELECT1	(3.1325,	0.32795)		(0.4123,	1.904)
SELECT2	(3.2718,	0.9807)		(0.3213,	1.3958)
SELECT3	(2.67155,	1.94215)		(0.64295,	0.94955)
SELECT4	(1.2047,	2.37125)		(1.27785,	0.7721)
SELECT5	(2.912,	1.85325)		(0.5306,	0.98665)
SELECT6	(0.8099,	1.1466)		(1.49695,	1.30725)

IV. DISCUSSION and CONCLUSION

The muscle fiber action potential was generated by transforming the muscle fiber position. To relocate fiber position, the Gaussian distribution was applied through Box Müller transformation. Based on the results of this simulation, it was possible to observe that the amplitude of each action potential increased by about 2 times, compared with that in the state of uniformly distribution. This implies that the applied distribution curve for muscle fiber location on the cross-section area affects the calculation of action potentials. Considering the importance of action potential in EMG, the increased amplitude of each action potential is expected to increase the amplitude of EMG. Generally, when the muscle contracts strongly, EMG shows high amplitude. Through this fact, the Gaussian distribution of muscle fibers can be considered to improve the EMG model, and it also helps the EMG generated by this method to match with real situation. Even though there are more factors to be considered in EMG simulation for adding more reality, the spread pattern of muscle fibers is the most important to generate EMG as mentioned. Therefore, the Gaussian distribution can be judged to play an efficient role in action potential and EMG simulation.

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