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Vectors and Tensors

They arise as generalizations of scalars and vectors that are common in Physics.

Tensors have a rank:

rank	notation	entity	examples
0	a	scalar	mass, energy
1	a_i	vectors	velocity, momentum
2	a_{ij}	matrix	polarization tensor, inertia tensor
3	a_{ijk}	cube	Hall effect coefficient
4	a_{ijkl}	hypercube	stress tensor

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The indices range from 1 to N where N is the dimension of the space.

N	examples
2	2-d space for graphene (x, y)
3	real space (x, y, z)
4	relativity (x, y, z, t)
$\vdots$	
m	? $(x_1, x_2, \ldots, x_n)$

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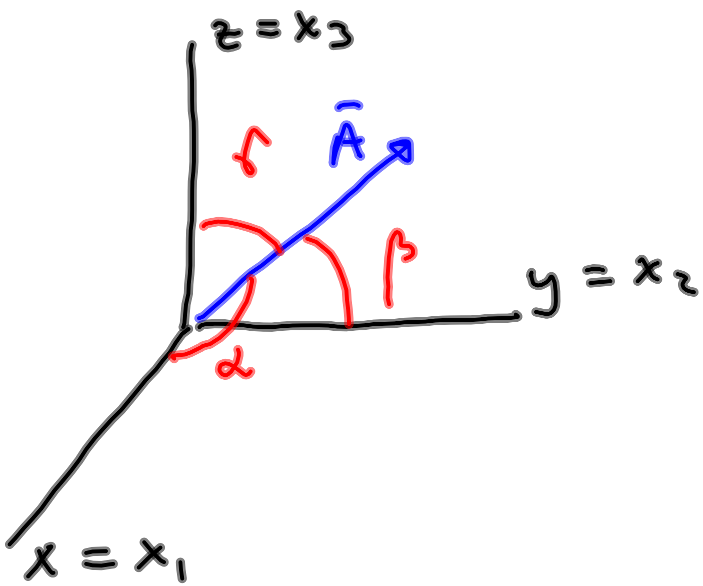
Vectors: tensors of rank 1

In $N=3$

System of reference:
cartesian coordinates



• magnitude $|\bar{A}|$

• direction (α, β, γ)

components:

$A_x = A_1 = |\bar{A}| \cos\alpha$

$A_y = A_2 = |\bar{A}| \cos\beta$

$A_z = A_3 = |\bar{A}| \cos\gamma$

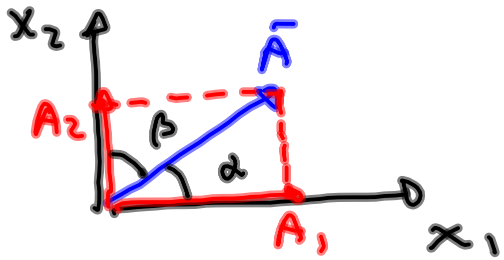
$$|\bar{A}| = (A_x^2 + A_y^2 + A_z^2)^{1/2} \quad (\text{Hw\#1})$$

$|\bar{A}|$ is an scalar it is a tensor of rank 0.

$\cos\alpha, \cos\beta, \cos\gamma$: direction cosines and $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$ (Hw#1).

Systems of Reference $N=2$

Cartesian system:



A_1 is the projection of $\bar{A}$ parallel to x_2 or perpendicular to x_1.

$\bar{A} = (A_1, A_2) \equiv A_i = A^i$

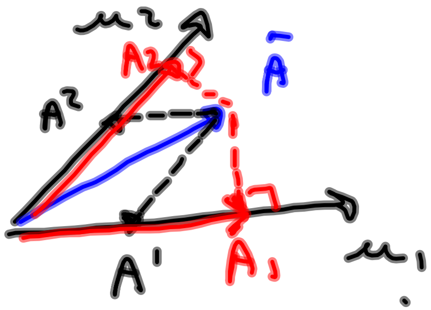
$A_1 = A\cos\alpha$

$A_2 = A\cos\beta = A\sin\alpha$

$|\bar{A}| = (A_1^2 + A_2^2)^{1/2}$

$\cos^2\alpha + \cos^2\beta = \cos^2\alpha + \sin^2\alpha = 1$

oblique system



$\bar{A} = (A^1, A^2) \equiv A^i$ (parallel projection)

$\bar{A} = (A_1, A_2) \equiv A_i$ (perpendicular projection)

Two sets of possible components.

$\left.\begin{array}{l} A_i : \text{covariant} \\ A^i : \text{contravariant} \end{array}\right\}$ to be shown later

Tensorial Definition of a vector.

We are going to define a vector in terms of the way in which its components get transformed as we change our system of reference.

For that we need to define a prototype vector.

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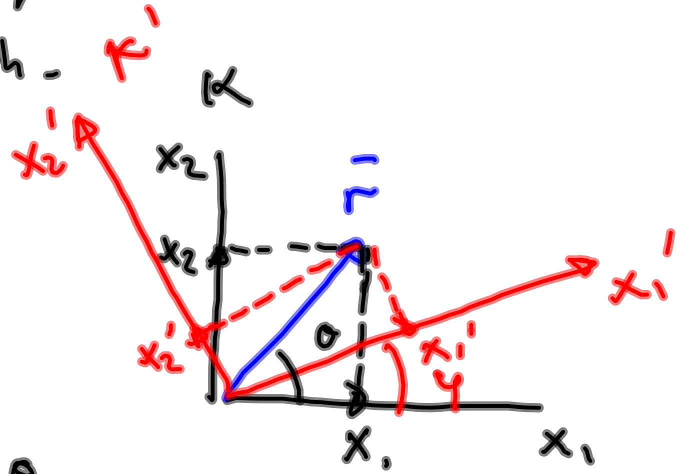
Prototype vector: $\bar{r}$ (position vector).

In physics vectors have units proportional to length.

$N = 2$



Find:

$x_1' = f(x_1, x_2)$

$x_2' = f(x_1, x_2)$

$x_1 = r\cos\theta$

$x_1' = r\cos(\theta - \varphi) =$

$= \underbrace{r\cos\theta}_{x_1}\cos\varphi + \underbrace{r\sin\theta}_{x_2}\sin\varphi =$

$= x_1\cos\varphi + x_2\sin\varphi$

$x_2 = r\sin\theta$

$x_2' = r\sin(\theta - \varphi) =$

$= \underbrace{r\sin\theta}_{x_2}\cos\varphi - \underbrace{r\cos\theta}_{x_1}\sin\varphi =$

$= x_2\cos\varphi - x_1\sin\varphi$

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