Midterm Exam II

P571

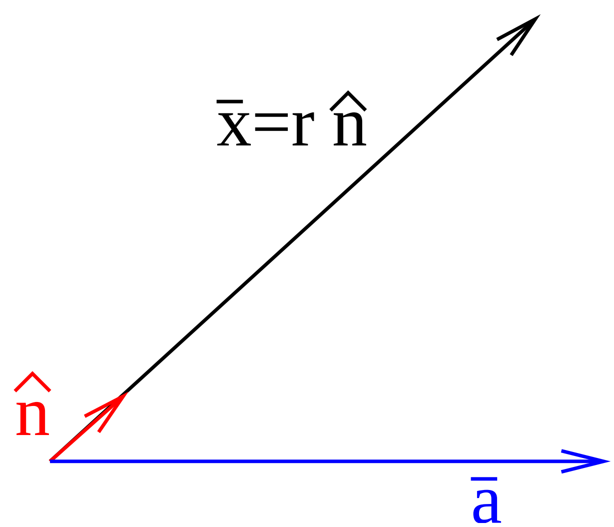
November 15, 2012

SHOW ALL WORK TO GET FULL CREDIT!

PART I:**ONLY ONE OF THE THREE PROBLEMS WILL BE GRADED**. Take a look at the 3 problems. Each of them is worth 25 points. To make sure that you have enough time to do your work you will have to turn in only **ONE** of the 3 problems. **If you turn more than 1 problem only the one on top will be graded and 5 points will be deducted from your grade.**

PART II: Take the test home and bring **ALL** the problems solved on **Tuesday November 20**. Your grade for the test will be the **sum of the two** parts. A perfect score is worth 100 points.

Problem 1: The figure shows a 3-dimensional vector $\mathbf{a} = (a_x, a_y, a_z) = a^i$ and a 3-dimensional vector $\mathbf{x} = (x, y, z) = r\hat{n} = x^i$ where $r = \sqrt{x^2 + y^2 + z^2} = (x_i x^i)^{1/2}$ and $\hat{n} = \mathbf{x}/r$ is a versor.



a) Express $\hat{n} \times \mathbf{a} = \mathbf{B}$ in tensor notation. Is $\mathbf{B}$ a tensor or a pseudotensor? Why? What is its rank?(5 points)

b) Using tensor notation show that

$$(\hat{n} \times \mathbf{a}) \times \hat{n} = \mathbf{a} - (\mathbf{a}.\hat{n})\hat{n}.$$

(5 points)

c) Draw $(\hat{n} \times \mathbf{a}) \times \hat{n}$ in a copy of the figure provided and indicate its geometrical meaning.(5 points)

d) Show that $\nabla.\hat{n} = 2/r$ using tensor notation. (5 points)

e) Now using tensor notation show that

$$r(\mathbf{a}.\nabla)\hat{n} = \mathbf{a} - (\mathbf{a}.\hat{n})\hat{n}.$$

(5 points)

Problem 2: The field strength tensor $F^{\alpha\beta}$ in frame S for a particle with charge q in uniform motion is given my

$$F^{\alpha\beta} = \frac{q}{c} \frac{(X^\alpha U^\beta - X^\beta U^\alpha)}{[\frac{1}{c^2}(U_\alpha X^\alpha)^2 - X_\alpha X^\alpha]^{3/2}},$$

where U^α and X^α are 4-vectors in Minkowski space that denote the 4-velocity and the position where the fields are being evaluated.

a) Is the denominator in the expression for $F^{\alpha\beta}$ given by $[\frac{1}{c^2}(U_\alpha X^\alpha)^2 - X_\alpha X^\alpha]^{3/2}$ a tensor? Why? (2 points)

b) Calculate $X_\alpha X^\alpha$ and $U_\alpha X^\alpha$ providing numerical values in terms of c when $X^\alpha = (1, 1, 0, 1/2)$ and $U^\alpha = c(1/2, \sqrt{3}/2, 1, 1)$. (3 points)

c) For the values of X^α and U^α given in part (b) provide the values of the 3 components of the electric and the magnetic fields, i.e., provide E_x, E_y, E_z, and B_x, B_y, B_z. (5 points)

d) Provide the value of $F^{\alpha\beta}F_{\alpha\beta}$ for the values of X^α and U^α given in part (b).(3 points)

e) Now consider a frame of reference S' with

$$M^\mu{}_\nu = \frac{\partial x'^\mu}{\partial x^\nu} = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

with $\beta = 1/3$ and $\gamma = 1.06$. Calculate X'^α and U'^α for the values of X^α and U^α given in part (b). (5 points)

f) Provide the value of E'_y in system S' at the values for X'^α and U'^α obtained in part (e) and compare it with the value of E_y found in part (c). Has it changed? Why? (5 points)

g) Will the result of part (d) be different in system S'? Why? (2 points)

Problem 3: A spherical shell of radius a centered at the origin has a surface potential given by $\Phi_a = V_0 \cos\theta$. The shell of radius a is inside a larger concentric shell with radius b ($b > a$). The shell with radius b is at potential $\Phi_b = 2V_0$.

a) In order to find the potential in the region in between the two spheres ($a \leq r \leq b$) what differential equation do you need to solve? (2 points)

b) What system of coordinates would you use to solve the differential equation? In terms of what functions will the potential be given? (3 points)

c) Now find the electrical potential $\Phi(\mathbf{r})$ inside the region in between the two concentric spherical shells with radius a and b with $a < b$ described in (a). Hint: check your result by verifying that it works at certain limits, for example, consider the limit $a \to 0$ and check that you result is what you expect. (10 points)

d) Find the electrical potential outside the spheres. (5 points)

e) Find the charge distribution on the external shell. (5 points)