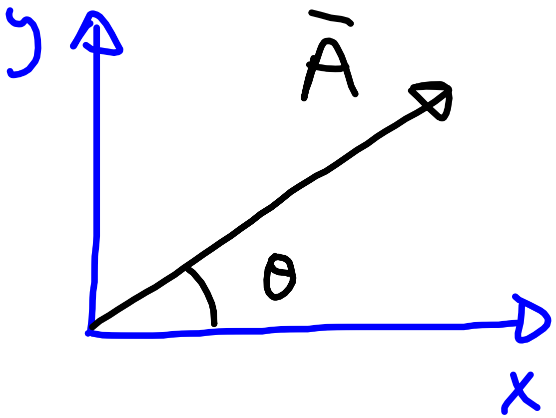
8/22

Vectors and Tensors:

(Read Ch. 1.7 for refresher on vectors).

Vectors:



$\bar{A}$ has magnitude $|\bar{A}|$ and direction θ.

A vector is also a tensor of rank 1 characterized as A_i.

- i runs from 1 to N where N is the dimension of the space.
- All the laws of Physics can be expressed in terms of tensors.

Tensors

rank	notation	name	Examples
0	a	scalar	mass charge speed
1	a_i	vectors	velocity force electric field
2	a_{ij}	matrix	moment of inertia cuadrupole
3	a_{ijk}	cube	octopole moment
4	a_{ijkl}	hypercube	stress tensor

multipole expansion:

$$q_0 = \int \rho(\bar{r})\, dV$$

$$p_i = \int \rho(\bar{r})\, r_i\, dV$$

$$q_{ij} = \int \rho(\bar{r})\, r_i r_j\, dV$$ quadrupole

$$q_{ijk} = \int \rho(\bar{r})\, r_i r_j r_k\, dV$$ octopole moment

Stress Tensor:

$$\varepsilon_{ab} = \sum_{c,d} \overbrace{S_{abcd}}^{\text{stress tensor (rank 4)}}\ \sigma_{cd}$$

strain (tensor of rank 2)

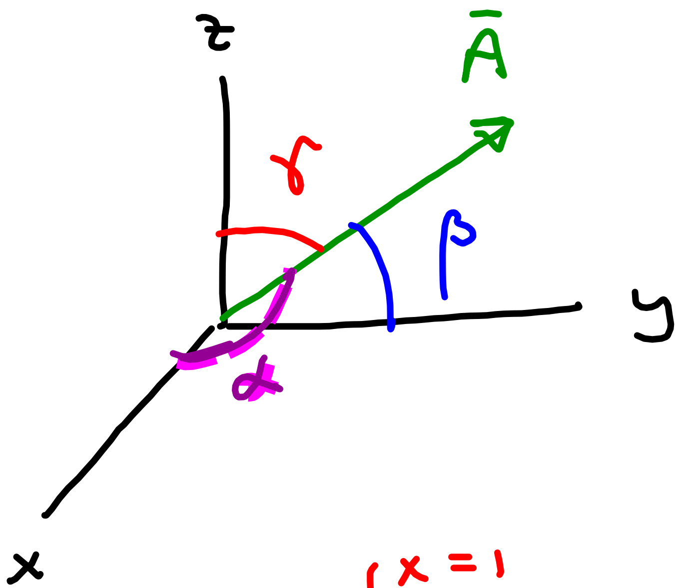
stress (tensor of rank 2)

A tensor of rank k has N^k components (N: space dimension)

—— x ——

Vectors

N = 3



Tensor notation: $\begin{cases} x = 1 \\ y = 2 \\ z = 3 \end{cases}$

$|\bar{A}|$: magnitude

$= [A_x^2 + A_y^2 + A_z^2]^{1/2}$

scalar

$A_x = |\bar{A}| \cos\alpha$

$A_y = |\bar{A}| \cos\beta$

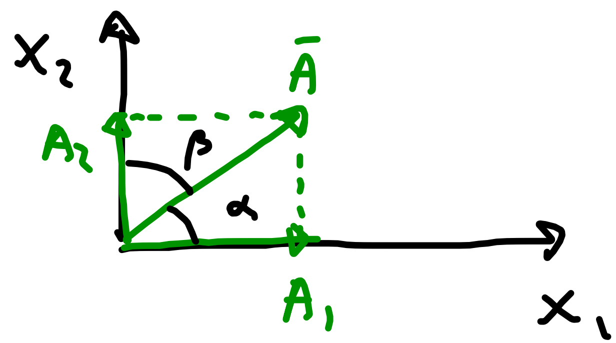
$A_z = |\bar{A}| \underbrace{\cos\gamma}_{\text{director cosines}}$

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$ Hw #1.

Systems of Reference

$N = 2$

Cartesian System



A_1 is the projection $\parallel$ to x_2 or $\perp$ to x_1.

$\bar{A} = (A_1, A_2) \equiv A_i \equiv A^i$

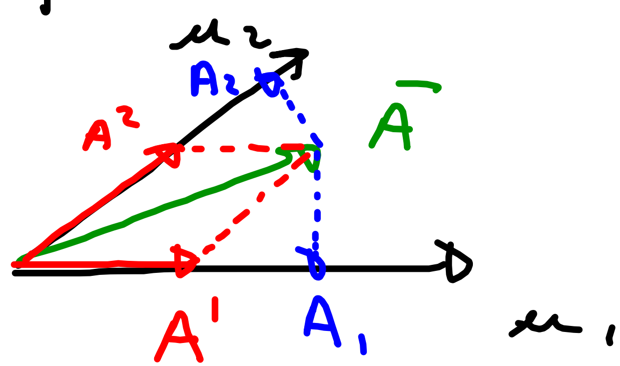
$A_1 = |\bar{A}| \cos\alpha$

$A_2 = |\bar{A}| \cos\beta$

$|\bar{A}| = [A_1^2 + A_2^2]^{1/2}$

$\cos^2\alpha + \cos^2\beta = 1$

Oblique System (crystals)



$\bar{A} = (A^1, A^2) \equiv A^i$ (parallel projection)

$\bar{A} = (A_1, A_2) \equiv A_i$ (perp. projection)

Two sets of components.

A_i : covariant
A^i : contravariant

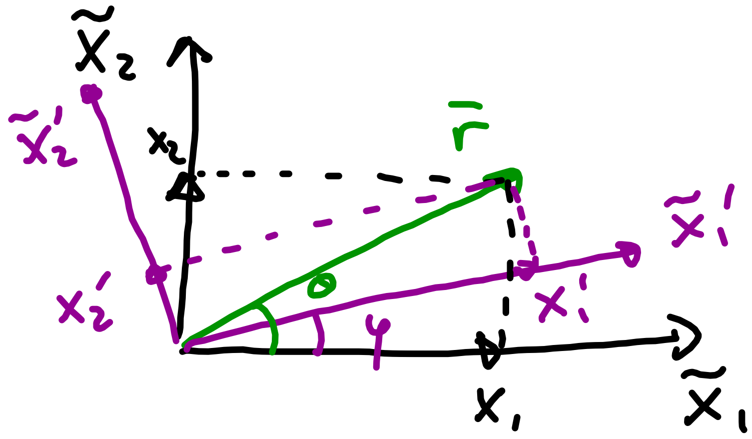
to be shown later

General definition of vector

A vector is an entity whose coordinates transform in a well-defined way from system K to K'.
The transformation is given by the behavior of a "prototype" vector

Prototype: vector position $\bar{r} \equiv r_i$

Prototype vector $\bar{r}$: $(N=2)$



Find:

$$x_1' = f(x_1, x_2)$$
$$x_2' = \tilde{f}(x_1, x_2)$$

$$x_1 = r\cos\theta$$

$$x_1' = r\cos(\theta-\varphi) =$$
$$\underbrace{r\cos\theta}_{x_1}\cos\varphi + \underbrace{r\sin\theta}_{x_2}\sin\varphi =$$
$$= x_1\cos\varphi + x_2\sin\varphi$$

$$x_2 = r\sin\theta$$

$$x_2' = r\sin(\theta-\varphi) =$$
$$\underbrace{r\sin\theta}_{x_2}\cos\varphi - \underbrace{r\cos\theta}_{x_1}\sin\varphi$$
$$= -x_1\sin\varphi + x_2\cos\varphi$$

Then we can write:

$$\begin{pmatrix} x_1' \\ x_2' \end{pmatrix} = \underbrace{\begin{pmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{pmatrix}}_{M} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

or

$$x_i' = \sum_{j=1}^{2} M_{ij} x_j \equiv M_{ij} x_j$$

Einstein's notation

Sum over repeated indices.