Midterm Exam

P571
October 3, 2013

SHOW ALL WORK TO GET FULL CREDIT!
WARNING!!! Points will be taken if numerical calculations are nor provided and if calculations are left just indicated.

PART I: **DO IT IN CLASS** Turn your work in before leaving. Take the printed copy of the test home.

PART II: Take the test home and bring **ALL** the questions solved on Tuesday October 8. Your grade for the test will be the **sum of the two** parts. Each question is worth 4 points. A perfect score is worth 80 points as a result of 20 points to be earned in class and 60 points to be earned at home. If you are 100% sure about the work you did in class, you do not need to redo it at home. In that case the points obtained in class will be counted twice.

## PART I

Consider a cartesian system $S$ in 3-dimensional space where the prototype contravariant vector is given by $\mathbf{r} = x^i \hat{\mathbf{e}}_i$ and a coordinate system S' also in three-dimensional space with axis $y'$ parallel to the $y$ axis in $S$, axis $z'$ parallel to the $z$ axis in $S$ and axis $x'$ making an angle $\alpha$ with the $x$ axis in $S$, an angle $\pi/2 - \alpha$ with the $y$ axis in $S$ and perpendicular to the $z$ axis (see the figure).

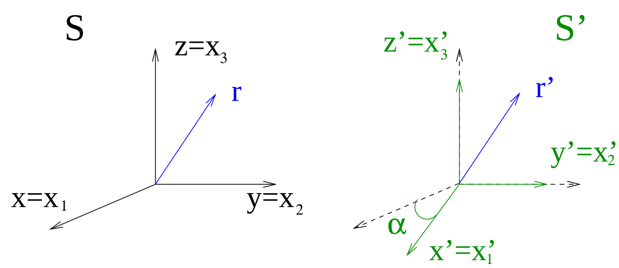


FIG. 1: The figure is approximated, i.e., not to scale which means that you cannot “read” the answers from the graph. Hint: making your own figure may be helpful.

a) Provide the coordinates $x'^i$ of the vector **r'** in $S'$ in terms of $x^j$, i.e.,the coordinates of $\mathbf{r}$ is $S$ and the angle $\alpha$. .

b) Using the results obtained in (a) write the transformation matrix $M^i{}_j = \frac{\partial x'^i}{\partial x^j}$.

c) Write expressions for the vectors $\hat{\mathbf{e}}_i$ that form the covariant basis in $S$.

d) Write expressions for the vectors $\hat{\mathbf{e}'}_i$ that form the covariant basis in $S'$. Provide expressions for them in the orthogonal system $S$.

e) Calculate the metric tensor $g'_{ij}$ in $S'$.

**STOP HERE!!!!**: Hand your work before leaving and take home the printed copy of the test. Bring **ALL** the questions answered on Tuesday October 8.

## PART II

f) Using the metric tensor and the contravariant components of **r'** found in (a) calculate the covariant components $x'_i$ of **r'**. Express you result in terms of the components $x^i$ of **r** in the cartesian system $S$.

Now consider that the angle between the axis $x$ and $x'$ is $\alpha = \pi/4$ and a vector **r**=(3,2,1) in $S$:

g) Provide the numerical values of the components $x'^i$ and $x'_i$ of **r** is $S'$.

h) Using the previous results evaluate $x'_i x'^i$ for **r'** in $S'$ and verify that it equals $x_i x^i$ for **r** in $S$.

Now consider the tensor $T'^{ij}{}_{kl} = r'^i r'^j r'_k r'_l$ in $S'$.

i) What is the rank of the tensor $T'$ and how many components does it have?

j) Write an expression for $T^{mn}{}_{op}$ in terms of $T'^{ij}{}_{kl}$ using the transformation rules from $S'$ to $S$. Note: Do not write each element of the tensor explicitely. Just provide the generic expression in terms of the matrix elements of the transformation.

k) Write the value of $T'^{12}{}_{21}$ and $T^{12}{}_{21}$ for **r**=(3,2,1). Do you expect the two to be equal? Why?

l) How many independent components does $T^{mnop}$ have? Why?

m) What is the rank of the tensor $T'^{ij}{}_{ij}$?

n) Provide the explicit numerical value of the tensor $T'^{ij}{}_{ij}$ for **r**=(3,2,1).

o) Provide the explicit numerical value of the tensor $T^{ij}{}_{ij}$ for **r**=(3,2,1) and compare it with $T'^{ij}{}_{ij}$. Did you expected this result? Why?