Midterm Exam II

P571

November 14, 2013

SHOW ALL WORK TO GET FULL CREDIT!
WARNING!!! Points will be taken if numerical calculations are not performed and if calculations are left just indicated.

PART I: **ONLY ONE OF THE THREE PROBLEMS WILL BE GRADED**. Take a look at the 3 problems. Each of them is worth 25 points. To make sure that you have enough time to do your work you will have to turn in only **ONE** of the 3 problems. **If you turn more than 1 problem only the one on top will be graded and 5 points will be deducted from your grade.**

PART II: Take the test home and bring **ALL** the problems solved on **Tuesday November 19**. Your grade for the test will be the **sum of the two** parts. A perfect score is worth 100 points.

Problem 1:

a) Using tensor notation show that for two vectors $\mathbf{A}$ and $\mathbf{B}$,

$$[(\mathbf{A}\times\mathbf{B})\times\mathbf{B}].\mathbf{A} = (\mathbf{A}.\mathbf{B})^2 - A^2B^2.$$

(10 points).
b) What is the rank of $[(\mathbf{A}\times\mathbf{B})\times\mathbf{B}].\mathbf{A}$? (5 points).
c) If $\mathbf{A}$ is a polar vector and $\mathbf{B}$ is an axial vector is $[(\mathbf{A}\times\mathbf{B})\times\mathbf{B}].\mathbf{A}$ a tensor or a pseudotensor? Why? (5 points).
d) If $\mathbf{A}\neq 0$ and $\mathbf{B}\neq 0$ under what conditions is $[(\mathbf{A}\times\mathbf{B})\times\mathbf{B}].\mathbf{A} = 0$? (5 points).

Problem 2: In a system S at rest the 4-vector potential is given by $A^\alpha = (\Phi, \mathbf{A}) = (-E_o x^1, -\frac{B_0}{2}x^2, \frac{B_0}{2}x^1, 0)$.
a) Find the field strength tensor $F^{\alpha\beta}$ in terms of E_0 and B_0. (5 points).
b) Give the values of $\mathbf{E} = (E_x, E_y, E_z)$ and $\mathbf{B} = (B_x, B_y, B_z)$ in system S in terms of E_0 and B_0. (5 points)
c) A general Lorentz transformation from a system S at rest to a system S' moving with velocity $\mathbf{v}$ with respect to S is given by

$$x'^0 = \gamma(x^0 - \vec{\beta}.\mathbf{x}),$$

$$\mathbf{x}' = \mathbf{x} + \frac{(\gamma-1)}{\beta^2}(\vec{\beta}.\mathbf{x})\vec{\beta} - \gamma\vec{\beta}x^0,$$

where $\vec{\beta} = \mathbf{v}/c$ and $\gamma = (1-\beta^2)^{-1/2}$. If $\mathbf{v} = (u, u, 0)$ provide the transformation matrix $M^\mu{}_\nu = \frac{\partial x'^\mu}{\partial x^\nu}$ in terms of γ, u, and c. (5 points).
d) Find A'^α in system S' in terms of E_0, B_0, u, c, x^i and γ. (5 points).
e) Find E'_y the y-component of the electric field $\mathbf{E}'$ in S' in terms of E_0, B_0,u, c, and γ. Hint: Calculate β^2 in terms of u and c and notice that $\beta^2 = \frac{\gamma^2-1}{\gamma^2}$.(5 points).

Problem 3:

a) Consider the following array of charges: a charge q at $(x, y, z) = (0, d, d)$, another charge q at $(x, y, z) = (0, -d, d)$, a charge $-q$ at $(x, y, z) = (0, d, -d)$, and another charge $-q$ at $(x, y, z) = (0, -d, -d)$.

i) Provide the location of each charge in spherical coordinates (r, θ, ϕ). (2.5 points).

ii) Write an expression for the electrostatic potential due to the 4 charges in terms of spherical harmonics $Y_{l,m}(\theta, \phi)$. (2.5 points).

iii) Use the symmetry properties of the spherical harmonics so that you can list all the values of l and m for which the coefficients in the expansion of the potential found in (ii) are zero. (2.5 points).

iv) Taking this into consideration, relabel your indices so that only the non-zero terms appear in your expression for the potential due to the charges. Note: if you already took care of this in part (ii) congratulations! Just repeat the result here. (2.5 points).

b) Now consider the limit $d \to 0$ with $qd = p$ where p is a constant, for the array of charges given in (a).

i) In this situation, what is the multipole moment that the array will have? On which ones of the variables r, θ, and ϕ do you expect that the potential should depend? Why? (2.5 points).

ii) How many terms remain in the expansion of the potential? Why? (2.5 points).

iii) Provide an expression for the potential in terms of p.(2.5 points).

c) Now the point-like array of charges whose potential you found in (b-iii) is placed at the center of a grounded spherical shell of radius a.

i) On what variables the electrostatic potential inside the spherical shell will depend? Why? (2.5 points).

ii) Propose an expression for the potential in terms of undetermined constants and indicate the boundary conditions that you will use to find the constants. (2.5 points).

iii) Now calculate the electrostatic potential inside the sphere. (2.5 points).