Homework #1

Problem 2 - 1.7.6:

Part a) Orient the system of coordinates so that the z-axis is parrallel to **a**. Then $\mathbf{a} = (0, 0, a)$ and $\mathbf{r} = (x, y, z)$ and

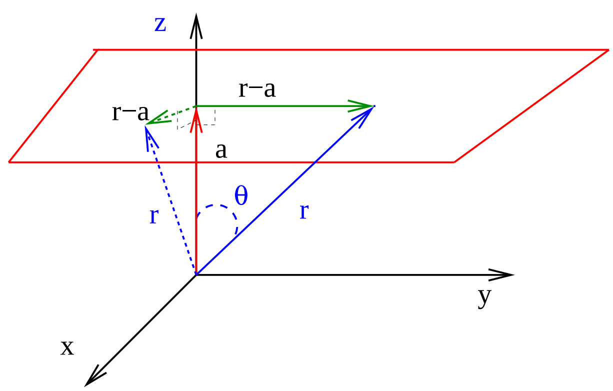
$$(\mathbf{r} - \mathbf{a}).\mathbf{a} = 0$$

$$(x, y, z - a).(0, 0, a) = 0$$

$$za = a^2.$$

$$z = a. \tag{1}$$

Eq.(1) indicates that the vectors **r** satisfying the constraint have the form $\mathbf{r} = (x, y, a)$ which means that the tip of **r** is on the horizontal plane defined by $z = a$ as shown in Fig.1.



Part b) As in part (a) let's orient the system of coordinates so that the z-axis is parrallel to **a**. Then $\mathbf{a} = (0, 0, a)$ and $\mathbf{r} = (x, y, z)$ and

$$(\mathbf{r} - \mathbf{a}).\mathbf{r} = 0 \tag{2}$$

$$(x, y, z - a).(x, y, z) = 0$$

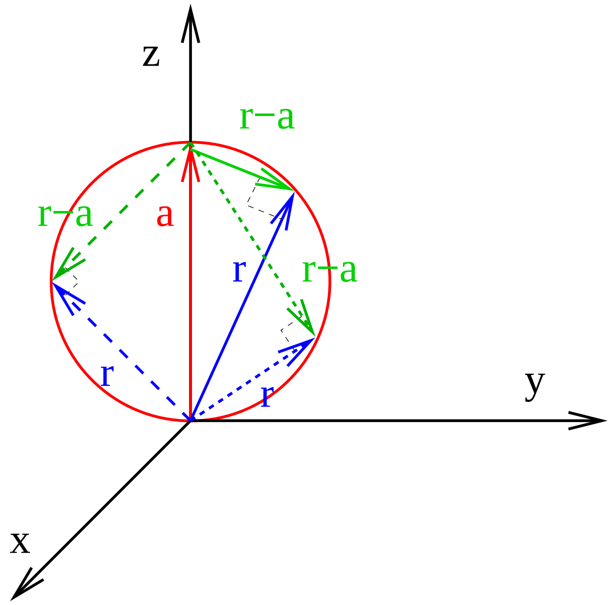
$$x^2 + y^2 + (z - a)z = 0. \tag{3}$$

Now let's add and substract $a^2/4$ in Eq.(3) in order to complete the square. We obtain

$$x^2 + y^2 + z^2 - za + \frac{a^2}{4} - \frac{a^2}{4} = 0,$$

$$x^2 + y^2 + (z - \frac{a}{2})^2 = (\frac{a}{2})^2. \tag{4}$$

Eq.(4) indicates that (x, y, z), the coordinates of **r** that satisfy Eq.(2), lie on a sphere of radius $\frac{a}{2}$ centered at the point $(0, 0, a/2)$ as shown in the figure.


z
r−a
r−a
a
r
r−a
r
r
y
x