Homework #10

Problem 1 - 15.4.10:

We need to calculate

$$\sin\theta \frac{dP_n(\cos\theta)}{d(\cos\theta)} \tag{1}$$

Doing the change of variables $x = \cos\theta$ we see that Eq.(1) becomes:

$$(1-x^2)^{1/2} \frac{dP_n(x)}{dx}. \tag{2}$$

Then, plugging $m = 1$ in Eq.(15.79) in the book, we obtain that

$$P_n^1(x) = -(1-x^2)^{1/2} \frac{dP_n(x)}{dx}. \tag{3}$$

Combining (3) and (2) we obtain

$$(1-x^2)^{1/2} \frac{dP_n(x)}{dx} = -P_n^1(x), \tag{4}$$

and changing back to the original variables (4) becomes:

$$\sin\theta \frac{dP_n(\cos\theta)}{d(\cos\theta)} = -P_n^1(\cos\theta). \tag{5}$$