Homework #10

Problem 2 - 15.4.13:

$$\int_0^\pi sin^2\theta P_n^1(\cos\theta)d\theta = \int_{-1}^1 \sin\theta P_n^1(\cos\theta)d(\cos\theta). \tag{1}$$

But we notice that $-\sin\theta = P_1^1(\cos\theta)$ and we can evaluate the integral using the orthogonality properties of the associate Legendre polynomials given in (15.104). Then:

$$\int_{-1}^1 \sin\theta P_n^1(\cos\theta)d(\cos\theta) = -\int_{-1}^1 P_1^1(\cos\theta)P_n^1(\cos\theta)d(\cos\theta) = -\frac{2}{2n+1}\frac{(n+1)!}{(n-1)!}\delta_{n,1} = -\frac{4}{3}\delta_{n,1}. \tag{2}$$