Homework #10

Problem 3 - 15.5.2:

From the definition of $Y_l^m(\theta,\phi)$ we know that

$$Y_l^m(0,\phi) = \sqrt{\frac{2l+1}{4\pi}\frac{(l-m)!}{(l+m)!}}P_l^m(1)e^{im\phi}. \tag{1}$$

Since

$$P_l^m(x) = (-1)^m(1-x^2)^{m/2}\frac{d^m P_n(x)}{dx^m}, \tag{2}$$

we see that if $m \neq 0$ and $x = 1$, Eq.(2) vanishes since in that case $(1-x^2)^{m/2} = 0$ and for $m = 0$ we know that $P_l(1) = 1$ for all l. Then

$$P_l^m(1) = (-1)^m\delta_{m0}. \tag{3}$$

Replacing (3) in (1) we obtain

$$Y_l^m(0,\phi) = (-1)^m\sqrt{\frac{2l+1}{4\pi}\frac{(l-m)!}{(l+m)!}}\delta_{m0}e^{im\phi} = \sqrt{\frac{2l+1}{4\pi}}\delta_{m0}. \tag{4}$$